Understanding the Relationship between the Volatility Risk Premium and Option Returns

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JEL classification: D83, G12, G13, G14, G17.
Keywords: Option returns, volatility risk premium, dynamic equilibrium model, rational learning, option pricing, implied volatility, implied volatility slopes.

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1 Introduction

Are option returns predictable? In spite of the enormous expansion of equity option markets since the first day of negotiations on the Chicago Board Option Exchange on April 26, 1973, there has been a limited number of efforts in the financial economics literature aimed at understanding the predictability features of option returns. Recently, Goyal and Saretto (2009) presented empirical evidence that the volatility risk premium (which can be measured as the difference between implied and realized volatilities) can predict option returns. Goyal and Saretto (2009) argue that large deviations of implied volatility (henceforth, IV) from realized volatility (henceforth, RV) are indicative of option mispricing, which could be used to predict returns on option contracts. We provide a different (although complementary) explanation for such empirical evidence. Through the use of a rather standard, but yet powerful, dynamic equilibrium model in which agents follow a rational learning process, we show that cognitive mechanisms explain the predictive relationship between the volatility risk premium and returns on option portfolios. Our option learning model is based on generally accepted assumptions concerning preferences and the stochastic process of the fundamentals which drive asset prices. Our findings complement the explanation given by Goyal and Saretto (2009), since we explain through rational learning 'why' IV can deviate from RV, but we also show that learning generates predictive dynamics. Moreover, our learning model is able to explain other empirical phenomena that have been observed in option market data, including the relationship between option returns and implied skewness (e.g., Bali and Murray, 2013), the association of option returns with the slope of the implied volatility term structure (e.g., Vasquez, 2012), and the relationship between predictive dynamics in implied volatilities and returns of option portfolios (e.g., Gonçalves and Guidolin, 2006; Bernales and Guidolin, 2013).

Our model extends the simple discrete-time endowment economy proposed by Lucas (1978), in which a representative agent has to price a risk-free one-period bond, a stock, and a set of option contracts (European put and call option contracts). We modify Lucas' (1978) model by making the fundamental mean dividend growth rate, g_t, subject to breaks, where time periods between breaks follow a memoryless stochastic process.[1,2] Thus, when a break takes place at time $t+m$, the new mean dividend growth rate is drawn from a continuous univariate density $g_{t+m} \sim G(\cdot)$ and its value keeps constant until the next break. Moreover, we relax the Lucas' (1978) full information assumption by assuming that the mean dividend growth rate is unknown by the representative agent. Nevertheless, the agent can learn recursively about the new value of g_{t+m} as new information arrives by following a Bayesian updating procedure.

In the model, the representative agent learns from the daily dividends received after each break; hence, dividends are the signals used to obtain an 'estimated' mean dividend growth rate, $\tilde{g}_{t+m}$, in order to price all assets. In early periods after a break, when no 'long' history of dividend realisations is available, there are dramatic revisions in the estimated $\tilde{g}_{t+m}$, since there is an important estimation uncertainty resulting from the lack of information. Thus, the initial uncertainty generated by the agent's learning process produce a change in the perception of the risk-neutral probability distribution which is seen as wider, and extreme events are perceived as more likely (see, e.g., Guidolin and Timmermann, 2003 and 2007). This induces a divergence between the risk-neutralized and physical volatility measures (i.e., typically measured through the implied and realized volatilities, respectively).[3] Therefore, learning could give an explanation to the volatility risk premium, which in our model is associated with the uncertainty generated by the agent's learning ability when there are changes in the fundamentals.[4,5]

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[1] There is an extended empirical literature showing evidence of breaks in economic fundamentals, such as in the parameters of the dividend process and in the real GDP growth (e.g., Bai *et al*., 1998; Timmermann, 2001; Granger and Hyung, 2004).

[2] Several circumstances can generate breaks in the economy, such as permanent technological innovations, shifts in tax codes, shifts in monetary policy, shifts in stock market participation, among other possibilities. However, as for any 'change', breaks in economic fundamentals induce a learning process that investors must follow to recognize the new market conditions.

[3] Thus, learning may provide an explanation for the puzzling empirical regularity that implied volatilities are typically greater than realized volatilities (see, e.g., Christensen and Prabhala, 1998).

[4] The volatility risk premium has been previously associated with economic uncertainties and business cycles (see Corradi *et al*., 2013; Todorov, 2010; Miao *et al*., 2012; Bollerslev *et al*., 2011).

[5] Several recent studies have proposed explanations for the existence of the volatility risk premium within the context of equilibrium-based pricing models. However, these studies have not explored why the volatility risk premium can predict option returns, which is the main objective of our study. See, e.g., Bakshi and Kapadia (2003a, 2003b), Bakshi and Madan (2006), Bollerslev *et al*. (2009), Carr and Wu (2009), Drechsler and Yaron (2011), Eraker (2009), and Todorov (2010).

The changes in the shape of risk-neutral probability distribution are directly observed in option prices and option returns, due to the non-linear features of the payoff structure of option contracts. Moreover, we show that learning has a heterogeneous impact on different option contracts (and hence an asymmetric effect on their implied volatilities), since their non-linear properties and leverage levels depend on their strike prices and time-to-maturities. Therefore, and similar to Guidolin and Timmermann (2003), we report that learning generates an implied volatility surface (henceforth, IVS).[6]

Nevertheless, learning is a dynamic and a recursive process. As more information becomes available, the estimated fundamental mean dividend growth ($\tilde{g}_{t+m}$) rate slowly converges to its true value (g_{t+m}); and hence the initial large values of the volatility risk premium and the impact of learning on option returns are progressively reduced. Thus, learning produces changes over time in the agent's beliefs and dynamically affects the risk-neutral probability distribution, which becomes path dependant. Consequently, learning generates dynamic impacts on asset valuations and affects other 'implied' variables obtained from option contracts. However, learning never disappears completely (even asymptotically) because its strength is destined to re-appear once again after a new break hits the mean dividend growth rate.

We find that the recursive learning process not only affect option prices and option returns, but learning also induces a dynamic relationship between option returns and the volatility risk premium (henceforth, VRP or (IV-RV)). We show through our model that the volatility risk premium consistently predicts a nontrivial fraction of returns of various option trading strategies (i.e., hold-to-maturity option returns, delta-hedged option portfolio returns and straddle portfolio returns). Thus, learning may provide an explanation for the empirical evidence documented in Goyal and Sarreto (2009) and Cao and Han (2013), who show that the volatility risk premium

[6] Empirical research has identified that implied volatilities tend to differ across strike prices and time-to-maturities, which is known as the implied volatility surface (see, e.g., Rubinstein,1985; Dumas *et al.*, 1998; Das and Sundaram, 1999).

has a predictive power in relation to economic profits obtained in portfolios with option contracts.[7,8]

Furthermore, we present evidence that the agent's learning process explains why the slopes of the IVS (on the moneyness dimension and on the maturity dimension) also predict option returns, which has been documented by previous empirical studies. For instance, Bali and Murray (2013) show that implied skewness, which is proxied by the implied volatility slope along the moneyness dimension, can forecast returns on option portfolios, while Vasquez (2012) reports that the implied volatility slope on the maturity dimension has a predictive power on option returns.[9] In addition, we show that learning also offers an explanation for the forecasting ability of implied volatilities over delta-hedged and straddle returns, that has been reported in index options (see Gonçalves and Guidolin, 2006) and equity options (see Bernales and Guidolin, 2013). Our results are robust across diverse trading strategies and after adding alternative predictor variables.

Our study is connected to equilibrium models in which learning is used in option pricing. Although these studies use learning mainly to explain the 'existence' of the implied volatility surface; hence they do not explore the predictability patterns and features of 'option returns' as our study does. For instance, David and Veronesi (2002) present a model in which the dividend drift follows a two-state regime-switching process, where investors have uncertainty about the current state of the economy. Guidolin and Timmermann (2003) introduce an equilibrium model where dividend growth evolves on a binomial lattice with an unknown but recursively updated state probability. Shaliastovich (2009) presents a long-run risk type model where the

[7] It is important to note that Goyal and Sarreto (2009) and Cao and Han (2013) use a negative version of our volatility risk premium, however our results are consistent with the evidence reported in both studies. Goyal and Saretto (2009) find that portfolios with a large positive difference between RV and IV produce an economically and statistically significant monthly option return. Cao and Han (2013) also show that the difference between RV and IV has a significant positive relationship with returns on call- and put-delta-hedged portfolios.

[8] Despite the fact that our model has only one stock and a set of option contracts, while Goyal and Sarreto (2009) and Cao and Han (2013) analyse option returns using diverse equity options, the main objective of our study is to show (using a simple extension of the Lucas' (1978) model) that learning may explain the puzzling relationship between the volatility risk premium and the returns of option portfolios.

[9] Toft and Prucyk (1997) use the implied volatility slope on the moneyness dimension of the IVS as a proxy for risk-neutral skewness.

expected consumption growth and its uncertainty are time-varying, and uncertainty is subject to jumps. In Shaliastovich's (2009) model, the unobservable consumption growth rate has to be learned by a recency-biased updating procedure. [10,11] However, we are unaware of any theoretical study to date that explains, through a dynamic equilibrium model, the *predictive power* on 'option returns' of: the volatility risk premium; the moneyness and maturity slopes of the IVS; and the 'simple' implied volatility.

We contribute to the body of knowledge on understanding why the volatility risk premium and other option-implied variables can forecast option returns. As previously stated, the use of a simple (but powerful) dynamic equilibrium model under rational learning to explain the predictability characteristics of option returns appears distinctive. The study is organized as follows. Section 2 introduces the model. Section 3 describes the simulations and Section 4 presents the main results. Finally, section 5 concludes.

2 The model

As mentioned in the introduction, there is empirical evidence that the difference between the implied and realized volatilities is able to explain and predict future option returns, which contradicts the assumptions of the Black and Scholes' (1973) model where the volatility risk premium should not exist. This suggests that a more general option pricing model is required. In this section, we propose an equilibrium model with learning to examine new and more complex linkages of option returns, volatility risk premium and other option-implied variables.

Our starting point builds on the simple representative agent discrete-time endowment economy introduced by Lucas (1978). However, the setup proposed in Lucas (1978) is extended by making the fundamental mean dividend growth rate, g_t, subject to

[10] In addition, there are associated studies in which the predictive power of the volatility risk premium is used to forecast returns of the underlying asset instead of options returns (e.g., Bollerslev, 2009; and Drechsler and Yaron, 2011).

[11] Our paper is also related to studies that examine and explain why the volatility risk premium increases with uncertainty, where this uncertainty is generated through learning (e.g., David and Veronesi, 2014) or through investors' disagreement (e.g., Buraschi *et al*., 2014).

breaks. In this section, as a first step, we assume full information to characterize asset prices (i.e., the agent knows the value of g_t over time). Afterwards, we relax the full information assumption to incorporate learning. Thus, we will assume that g_t is unknown. Nevertheless, as new information arrives, a representative Bayesian agent recursively learns about the level of the fundamental mean dividend growth rate

To begin, suppose there are four asset types: a one-period zero-coupon default free bond, B_t, in zero net supply; a stock with net supply normalized at one, S_t; a set of put option contracts, $Put_t(K,\tau)$, and a set of call option contracts, $Call_t(K,\tau)$, which are European-style with underlying asset S_t, strike price K, and time-to-maturity τ. We consider a perfect capital market with the objective of pricing assets. There are no taxes, no transaction costs, unlimited short sales possibilities, perfect liquidity, and no borrowing and lending constraints. The representative agent has a power utility at time t:

$$u(C_t) = \begin{cases} \dfrac{C_t^{1-\alpha} - 1}{1-\alpha} & \alpha \geq 0 \\ \ln C_t & \alpha = 1 \end{cases} \tag{1}$$

where C_t is the real consumption and α is the coefficient of relative risk aversion.

The stock pays out infinite real dividends, D, which evolve following a geometric random walk, $\ln(D_{t+1}/D_t) = \mu_{t+1} + \sigma\varepsilon_{t+1}$, with volatility σ and drift μ_{t+1} in which the innovation term, ε_{t+1}, is homoscedastic and serially uncorrelated. However, the fundamental mean dividend growth rate g_{t+1} (and hence μ_{t+1} given that $g_{t+1} = \exp(\mu_{t+1} + \sigma^2/2) - 1$) presents breaks and thus changes over time, although the value of g_{t+1} is constant between breaks. Time periods between breaks follow a geometric process with a parameter π; hence the number of breaks in a given period is characterized by a Binomial distribution. We assume that σ and π are constant to obtain the simplest setting which allows us to isolate the sources of learning to only one parameter (in our case, learning only involves the fundamental mean dividend growth rate). This is in line with Timmermann (1996, 2001), who shows that: i) this specification provides an adequate fit to the data on real dividends in the U.S. market; and ii) investors' learning regarding only the mean dividend growth rate induces excess volatility and volatility clustering in stock returns, even though the volatility of the dividend random walk process, σ, and the probability of breaks, π, are assumed to be invariable.[12]

Let us assume that when a break happens at time $t+m$, the new value of g_{t+m} is drawn from a continuous univariate density $g_{t+m} \sim G(\cdot)$ defined on the support $[g_d, g_u]$. We assume that dividends are non-storable and represent the unique source of income; thus they are consumed when they are received (i.e., $C_t = D_t$). Therefore, subject to budget constraints, the representative agent chooses assets' holdings by maximizing her discount value of expected future utilities, $\max_{\{D_{t+k}, w_{t+k}^S, w_{t+k}^B\}} E_t[\sum_{k=0}^{\infty} \beta^k\, u(D_{t+k})]$, in which $\beta = 1/(1+\rho)$, ρ is the impatience rate, and w_{t+k}^S (w_{t+k}^B) is the quantity of stocks (bonds) in her portfolio. As in Guidolin and Timmermann (2003), we assume that markets are complete and hence options are not considered in the agent's holding maximization problem since option contracts are redundant assets. Consequently, this yields the following Euler equations for the stock and the bond: $S_t = \mathrm{E}_t[\beta(D_{t+1}/D_t)^{-\alpha}(S_{t+1} + D_{t+1})]$ and $B_t = \mathrm{E}_t[\beta(D_{t+1}/D_t)^{-\alpha}]$, in which the pricing kernel, $Q_{t+1} = \beta(D_{t+1}/D_t)^{-\alpha}$, is defined as the intertemporal marginal rate of substitution multiplied by the discount factor. Proposition I presents expressions for equilibrium stock and bond prices, which are obtained by solving the Euler equations in the presence of breaks and full information.

<u>Proposition I (Full Information)</u>: The full information rational expected stock price, S_t^{FI}, and the one-period zero-coupon expected bond price, B_t^{FI}, are given by:

$$S_t^{FI} = \frac{D_t}{1+\rho-(1-\pi)(1+g_t)^{1-\alpha}} \left\{(1-\pi)(1+g_t)^{1-\alpha} + \pi\left(\frac{I_1+(1-\pi)I_2}{1-\pi I_3}\right)\right\} = D_t\Psi(g_t) \qquad (2)$$

and

$$B_t^{FI} = \frac{1}{(1+\rho)}\left\{(1-\pi)(1+g_t)^{-\alpha} + \pi\int_{g_d}^{g_u}(1+g_{t+1})^{-\alpha} dG(g_{t+1})\right\}, \qquad (3)$$

where $I_1 = \int_{g_d}^{g_u}(1+g_{t+1})^{1-\alpha} dG(g_{t+1})$; $I_2 = \int_{g_d}^{g_u} \frac{(1+g_{t+1})^{2-2\alpha}}{1+\rho-(1-\pi)(1+g_{t+1})^{1-\alpha}} dG(g_{t+1})$; $I_3 = \int_{g_d}^{g_u} \frac{(1+g_{t+1})^{1-\alpha}}{1+\rho-(1-\pi)(1+g_{t+1})^{1-\alpha}} dG(g_{t+1})$*; and* $1+\rho > (1+g_u)^{1-\alpha}$.

Proof: See Timmermann (2001).

[12] Appendix B shows that the geometric random walk with breaks in the drift is capable to characterize the dynamics of dividends using market data.

Proposition I shows that under full information, the ex-dividend stock price is first order homogeneous in dividends and is affected by shifts in g_t; thus, the price-dividend ratio is time-varying and also conditioned by g_t. Similarly, the bond price is also dynamic and evolves over time depending on the breaks in g_t. Moreover, assuming no arbitrage opportunities, option contracts can be priced by a change of measure in relation to the state-price density, which are presented in Proposition II.

<u>Proposition II (Full Information)</u>: The full information rational expected prices of the European put option, $Put_t^{FI}(K,\tau)$, and the European call option, $Call_t^{FI}(K,\tau)$, with strike price K and maturity τ are given by:

$$Put_t^{FI}(K,\tau) = \int_0^{\infty} \max\{K - S_{t+\tau}^{FI}, 0\}\, \tilde{p}_t(S_{t+\tau}^{FI}) dS_{t+\tau}^{FI} \quad (4)$$

and

$$Call_t^{FI}(K,\tau) = \int_0^{\infty} \max\{S_{t+\tau}^{FI} - K, 0\}\, \tilde{p}_t(S_{t+\tau}^{FI}) dS_{t+\tau}^{FI} \quad (5)$$

where $S_{t+\tau}^{FI} = D_{t+\tau}\Psi(g_{t+\tau})$, $D_{t+\tau} = D_t \exp\left(\sqrt{\tau}\sigma\varepsilon_{t+\tau} - \tau\sigma^2/2\right) \prod_{i=1}^{z}(1+g_{t+h_i})^{h_i}$, z is the number of breaks between t and $t+\tau$ that is a random variable drawn from a Binomial distribution $\varphi(z|\tau,\pi)$ with parameters τ and π, and $\{h_i\}_{i=1}^{z}$ are the time periods between breaks and also random variables drawn from geometric distributions $\eta(h_i|\pi)$ in which $\tau = \sum_{i=1}^{z} h_i$. In addition, $\{g_{t+h_i}\}_{i=2}^{z}$ are drawn from a univariate density $g_{t+h_i} \sim G(\cdot)$ with pdf $\varrho(\cdot)$ defined on the support $[g_d, g_u]$ where $g_{t+h_1} = g_t$ and $g_{t+h_z} = g_{t+\tau}$, $\varepsilon_{t+\tau}$ is the innovation term of the dividends' geometric random walk characterised by a normal density $\phi(\varepsilon_{t+\tau}|0,\sigma)$ with mean zero and volatility σ, and finally: $\tilde{p}_t(S_{t+\tau}^{FI}) = \beta^{\tau}(D_{t+1}/D_t)^{-\alpha}\phi(\varepsilon_{t+\tau}|0,\sigma)\varphi(z|\tau,\pi)\eta(h_1|\pi)\cdot\left[\eta(h_2|\pi)\varrho(g_{t+h_2}) \ldots \eta(h_z|\pi)\varrho(g_{t+h_z})\right]$.

Proof: See Appendix A.

Proposition I and Proposition II are obtained assuming full information (i.e., at any time t the representative agent knows the true fundamental value of the mean dividend growth rate g_t). However, we will relax this assumption to observe the effect of learning on option prices and their returns. Suppose that g_t is unknown (and therefore μ_t is also unknown given that $1 + g_t = \exp(\mu_t + \sigma^2/2)$); however, the representative agent observes the dividends received from the underlying asset on a daily basis. Thus, the agent receives independent signals about the mean dividend growth rate which are random and follow a lognormal distribution, $\{D_i/D_{i-1}\}_{i=t-n+1}^{t}$, where n is the number of periods since the last break. We assume that the representative agent uses the information available efficiently to price all assets following a Bayesian updating procedure. Similar to Timmermann (2001), we assume that the agent knows when a break happens, which allow us to study the clean effect of breaks and learning on option pricing. The assumption of knowing the breakpoint dates does not appear to be completely unrealistic, given the number of recent econometric advances that have shown that it is possible to perform real-time tests to monitor breaks in the mean function, and attaining a considerable degree of accuracy (see, e.g., Chu *et al*., 1996; Leisch *et al*., 2000).[13]

Consequently, given the agent's prior beliefs $f(\mu_t)$ when there is incomplete information, the expected value under Bayesian learning at time t, $\mathrm{E}_{t,n}^{BL}[\cdot]$, of any asset or variable that depends of μ_t, $\lambda_t(\mu_t)$, is:

$$\mathrm{E}_{t,n}^{BL}[\lambda_t(\mu_t)|\boldsymbol{\xi}_t] = \frac{\int_{\mu_d}^{\mu_u} \lambda_t(\mu_t) L(\mu_t|\boldsymbol{\xi}_t)_n f(\mu_t) d\mu_t}{\int_{\mu_d}^{\mu_u} L(\mu_t|\boldsymbol{\xi}_t)_n f(\mu_t) d\mu_t} \tag{6}$$

with

$$L(\mu_t|\boldsymbol{\xi}_t)_n = \frac{1}{\sqrt{2\pi\sigma^2/n}} \exp\left[\frac{-(\bar{\xi}_t - \mu_t)^2}{2\sigma^2/n}\right] \tag{7}$$

in which $\boldsymbol{\xi}_t = [\ln(D_t/D_{t-1}) \ldots \ln(D_{t-n+1}/D_{t-n})]$, $\bar{\xi}_t = (1/n)\sum_{i=t-n+1}^{t} \xi_i$, and $f(\cdot)$ is the pdf of μ_t.

Corollary I (Bayesian Learning): The stock price, the bond price, the prices of European put option contracts and the prices of European call option contracts under incomplete information and learning, are given by:

$$S_t^{BL} = \frac{\int_{\mu_d}^{\mu_u} S_t^{FI} L(\mu_t|\boldsymbol{\xi}_t)_n f(\mu_t) d\mu_t}{\int_{\mu_d}^{\mu_u} L(\mu_t|\boldsymbol{\xi}_t)_n f(\mu_t) d\mu_t} \quad , \tag{8}$$

[13] Moreover, as argued by Lettau and Van Nieuwerburgh (2008), the uncertainty generated by the detection of breakpoint dates in the process of economic fundamentals is not critical to explain asset returns anomalies. Lettau and Van Nieuwerburgh (2008) show that the main source of uncertainty is caused by the estimation of the magnitude of the new parameters in the aftermath of the break dates, similarly to our modelling approach.

$$B_t^{BL} = \frac{\int_{\mu_d}^{\mu_u} B_t^{FI} L(\mu_t|\boldsymbol{\xi}_t)_n f(\mu_t) d\mu_t}{\int_{\mu_d}^{\mu_u} L(\mu_t|\boldsymbol{\xi}_t)_n f(\mu_t) d\mu_t}\ , \tag{9}$$

$$Put_t^{BL}(K,\tau) = \frac{\int_{\mu_d}^{\mu_u} \left\{\int_0^\infty \max\{K - S_{t+\tau}^{FI}, 0\} \tilde{p}_t(S_{t+\tau}^{FI}) dS_{t+\tau}^{FI}\right\} L(\mu_{t+\tau}|\boldsymbol{\xi}_{t+\tau})_{n_{t+\tau}} f(\mu_{t+\tau}) d\mu_{t+\tau}}{\int_{\mu_d}^{\mu_u} L(\mu_{t+\tau}|\boldsymbol{\xi}_{t+\tau})_{n_{t+\tau}} f(\mu_{t+\tau}) d\mu_{t+\tau}}, \tag{10}$$

$$Call_t^{BL}(K,\tau) = \frac{\int_{\mu_d}^{\mu_u} \left\{\int_0^\infty \max\{S_{t+\tau}^{FI} - K, 0\} \tilde{p}_t(S_{t+\tau}^{FI}) dS_{t+\tau}^{FI}\right\} L(\mu_{t+\tau}|\boldsymbol{\xi}_{t+\tau})_{n_{t+\tau}} f(\mu_{t+\tau}) d\mu_{t+\tau}}{\int_{\mu_d}^{\mu_u} L(\mu_{t+\tau}|\boldsymbol{\xi}_{t+\tau})_{n_{t+\tau}} f(\mu_{t+\tau}) d\mu_{t+\tau}}, \tag{11}$$

Here, $S_{t+\tau}^{FI} = D_{t+\tau}\Psi(g_{t+\tau})$ *with* $g_{t+\tau} = \exp(\mu_{t+\tau} + \sigma^2/2) - 1$, *and* $n_{t+\tau}$ *is the number of signals at the maturity from the last break* (*it is important to observe that* $n_{t+\tau} \neq n + \tau$ *because there are chances of breaks between* t *and* $t + \tau$*; and hence* $n_{t+\tau}$ *is also a random variable where* $n_{t+\tau} \leq n + \tau$).

Therefore, immediately after breaks, the agent does not have enough historical information to obtain reliable values for the different asset prices. She experiences an initial period of intense learning that generates important changes in her beliefs about the fundamental mean dividend growth rate, which induce important effects on the valuation process of all assets, and has a particularly dynamic impact on option prices and their implied volatilities. However, these large adjustments in beliefs are reduced recursively over time, as more information is received and learned. In fact, as we will show in the following sections, the learning process by which the agent's beliefs are updated (i.e. according to a recursive information acquisition) produces predictive dynamics on option returns, volatility risk premium and other option-implied variables.[14]

3 Simulations

The main aim of our study is to understand the effects of learning on the relationship and predictability patterns of option returns, volatility risk premium and other

[14] It is important to notice that when the probability of a break is very large (i.e., $\pi \to 1$, which means that the agent faces breaks every day) learning is not observable at all, since everything changes constantly and 'there is no time to learn'. Whereas learning will vanish after a while in the case of an economy with no breaks (i.e., $\pi = 0$), even under incomplete information. When there are no breaks, the agent should have sufficient information after a long period to calculate accurate values for the mean dividend growth rate and asset prices; and thus learning will disappear asymptotically.

option-implied variables. Using the same arguments as in Timmermann (1993, 1996, 2001), Veronesi (1999, 2000) and Guidolin (2006), who all examine the effects of learning on stock returns by performing extensive sets of simulations, we use a simulation approach to analyse the learning effects on predictability patterns of option returns. Timmermann (1993, 1996, 2001), Veronesi (1999, 2000) and Guidolin (2006) argue that learning influences the investors' pricing in a highly nonlinear way and hence a simulation analysis is necessary to understand the wide scope of outcomes that learning generates in asset pricing. Moreover, a simulation approach allows us to adjust parameter setups and thus to analyse the effects of learning on multiple scenarios.[15]

We generate simulations from an economy with breaks and incomplete information under Bayesian learning, as described in the previous section. In each combination of parameters for the model, we generate 2,000 simulations. For each of these simulations, we produce 12 years (3,024 trading days) of daily dividends, which are the signals observed by agents to learn about g_t (which represents 6,048,000 simulated trading days). The simulations are generated by two nested stochastic processes. Firstly, we simulate time series of 12 years of daily dividends using the dividend's geometric random walk process ($\ln(D_{t+1}/D_t) = \mu_{t+1} + \sigma\varepsilon_{t+1}$). Secondly, we also induce breaks in g_t in each simulation (and hence breaks in μ_t), in which periods between breaks follow a geometric process with parameter π. For instance, in the case in which a break occurs at time $t+m$, we obtain a new value for g_{t+m} drawn from the univariate density $g_{t+m} \sim G(\cdot)$ defined on the support $[g_d, g_u]$, and we keep this value constant until the next break.

The stock prices and bond prices in each simulation are daily calculated using Equation (8) and Equation (9), respectively. European option prices are calculated monthly using Equation (10) and Equation (11), which are obtained through

[15] In addition, Kleidon (1986) presents evidence that the use of standard tests to analyse an equilibrium model in a single economy represented by market data could induce erroneous analysis. Kleidon (1986) points out that asset prices in equilibrium are calculated based on agents' expectations about future events across multiple and different economies; thus Kleidon (1986) also suggests a simulation approach.

numerical methods using Monte Carlo simulations based on 20,000 independent paths following the stochastic process described in Proposition II.[16,17]

We calculate option returns for a put contract and a call contract, both at-the-money and with one-month to maturity. Option returns are calculated through a hold-to-expiration trading strategy. We compute monthly option returns following a procedure akin to Ni (2009) and Broadie *et al*. (2009). The methodology consists in constructing time-series of returns with non-overlapping intervals. Put and call hold-to-maturity returns are defined, respectively, by the following equations:

$$r_{t+\tau}^{p} = \frac{\max(K - S_{t+\tau}, 0)}{p_t(K,\tau)} - 1 \tag{12}$$

and

$$r_{t+\tau}^{c} = \frac{\max(S_{t+\tau} - K, 0)}{c_t(K,\tau)} - 1, \tag{13}$$

where $c_t(K,\tau)$ and $p_t(K,\tau)$ are the prices of call and put options written at time t, with K as the strike price and τ as the time to maturity, while $S_{t+\tau}$ is the price of the stock at maturity $t+\tau$.

We also calculate the returns on a put-delta-hedged portfolio, a call-delta-hedged portfolio and a straddle portfolio. The put-delta-hedged portfolio (call-delta-hedged portfolio) is formed by buying one at-the-money one-month to maturity put contract (call contract) and buying (short-selling) delta shares of the stock, where the delta is obtained from the Black and Scholes' (1973) model. The straddle portfolio is calculated as a combination of buying one call option contract and one put option contract, where both contracts are at-the-money one-month to maturity. Similar to option returns, delta-hedged portfolios (for call and put options) and the straddle portfolio are obtained using a hold-to-expiration trading strategy on a monthly basis and also avoiding overlapping intervals.

[16] Note that these 20,000 paths in the Monte Carlo simulation are used to calculate option prices (with Equation (10) and Equation (11)) in each month of the 2,000 12-year time-series simulations described above.

[17] In the Monte Carlo simulations we also calculate the expected dividend yield and expected zero coupon interest rates for the time-to-maturity of each option contract with the objective of calculating later implied volatilities.

Following academic empirical studies and investors practices (see, e.g., Guidolin and Timmermann, 2003; and Gonçalves and Guidolin, 2006), we obtain implied volatilities by inverting the Black and Scholes' (1973) model. It is evident that the assumptions of the Black and Scholes' (1973) model are not followed in our modelling setup. However, researchers and practitioners also calculate implied volatilities using the Black and Scholes' (1973) model even though they know that its assumptions are violated in the reality by market data. In fact, the empirically observable relationship and predictability patterns between assets and the volatility risk premium, which we want to explain through our Bayesian learning model, have been observed with Black and Scholes' (1973) implied volatilities.

We assume the following plausible parameter values to be used in the simulations. We calculate asset prices using multiple levels for the coefficient of relative risk aversion at 0.2, 0.5 and 5.0. The rate of impatience, ρ, is set at 0.713% (monthly basis); while the new mean dividend growth rate after breaks is extracted from a uniform distribution defined between $g_u = 0.705\%$ and $g_d = -0.126\%$ on a monthly basis and thus $1 + \rho > (1 + g_u)^{1-\alpha}$, which are in line with the values used in Timmermann (2001).[18] The dividend process volatility, σ, is set at 0.014% on a monthly basis (i.e., 5% on an annual basis), which is also consistent with Timmermann (2001). Moreover, we use the test introduced by Chu *et al.* (1996) with real market data to obtain a value for the probability of breaks, π, on the mean dividend growth rate. Chu *et al.* (1996) present a dynamic test for structural breaks where market participants can contemporaneously identify a break on a given date due to the real-time features of their algorithm. We perform this test to calculate π using daily dividend time series from the S&P 500 index between 1996 and 2007 (which were deseasonalized and adjusted by the consumer price index to obtain real dividends as in Shiller, 2000). We find eight breaks in the 3,024 days of the 12 years analysed; and thus we set π at 0.67 (annual basis). In Appendix B, we describe the model introduced by Chu *et al.* (1996) and the breaks detected.

[18] Therefore, and given that new mean dividend growth rates after breaks are extracted from a uniform distribution with probability density function $f(g_t) = 1/(g_u - g_d)$, the dividend drift has as a probability density function: $f(\mu_t) = \exp(\mu_t + \sigma^2/2)/(g_u - g_d)$, where $\mu_d = \ln(1 + g_d) - \sigma^2/2$ and $\mu_u = \ln(1 + g_u) - \sigma^2/2$.

Furthermore, similar to Goyal and Saretto (2009), Bollerslev (2009) and Cao and Han (2013), we calculate the volatility risk premium as the difference between the option implied volatility and the realized volatility (i.e., $IV - RV$ which is a simple measure for the difference between the risk neutralized and physical volatilities). The IV is calculated as the average of the implied volatilities of a European put option contract and a European call option contract, where both contracts are at-the-money and with one-month to maturity. The RV is the standard deviation (annual basis) of the daily stock log-returns in each month (also avoiding overlapping periods).

In addition to the volatility risk premium, we also consider a set of other traditional predictor variables. We use as predictor variables IV and RV each of them alone (and calculated as described above). We also calculate the IV slope on the moneyness dimension, $Slope_{Mon}$, which is obtained as the difference between the IV from contracts with $K/S = 0.96$ and one-month to maturity (i.e., the average of the call and put contracts) and the IV from contracts with $K/S = 1.04$ and one-month to maturity (i.e., also the average of the call and put contracts). Furthermore, we calculate the IV slope on the maturity dimension, $Slope_{Mat}$, which is computed as the difference between the IV from at-the-money contracts with one-month to maturity (i.e., the average of the call and put contracts) and the IV from at-the money contracts with three-months to maturity (i.e., also from call and put contracts).

The existence of breaks in the mean dividend growth rate and the need of investors to learn about such an unstable time-varying parameter cause non-stationarities in option prices. To get some intuition for the nature of the instabilities captured by our framework, Figure 1 displays one complete simulation path in terms of the simulated mean dividend growth rate (g) and at-the-money short-term implied volatilities (IV_t). On the left hand side of the figure, we plot two time series: the time series of the true and estimated mean dividend growth rate (i.e., the estimated value of g is obtained over time by a rational investor who learns using the Bayes' rule presented in equation (6)). Looking at this time series, one can notice that learning may occasionally take a long time. Estimates of g progressively adjust toward the true values after each break, which is characterized by two effects. First, the observable dividend signals received by the investor are noisy because of the presence of the innovation term in the geometric random walk process. Consequently, the agent needs time to learn and thus to obtain accurate values for the unknown g. Second, in the figure a new break often appears when learning has improved the precision of the agent's estimations, and hence her accuracy regarding the estimated value of g is reduced and a strong learning process starts once again.

[Insert Figure 1 here]

On the right hand side of Figure 1, we reports the evolution of the implied volatility when there is learning (for the same simulation reported on the left hand side of this figure). This plot shows that the occasionally intense revisions of agents' expectations about the (new, post-break) value of g_t induce an increase in implied volatilities, especially in the immediate aftermath of breaks, when the learning speed accelerates and revisions are stronger. Figure 1 also shows that the average level of implied volatilities decreases as more information is received after each break. Different from earlier papers, such as Guidolin and Timmermann (2003), the effects of learning never disappear altogether, which is actually what happens in option markets. Thus, Figure 1 helps emphasizing that the interaction between learning and breaks may permanently and dynamically affect both the level and the evolution of implied volatilities.

Basic summary statistics for the simulations generated through our model are given in Table 1. In Table 1 and in the rest of the paper we will present only results for the returns on put options, the returns on the put-delta-hedged portfolio, and the returns on the straddle portfolio. Call option returns and the returns on the call-delta-hedged portfolio are unreported since the outcomes with call option contracts are quantitatively and qualitatively similar to the results presented here, and also congruent with the empirical literature. Nevertheless, the analysis using returns on call options and on the call-delta-hedged portfolio are available from the authors upon request.

[Insert Table 1 here]

In Table 1, the mean excess returns on the put option are equal to -55%, -39%, and –96% for coefficients of relative risk aversion at 0.2, 0.5, and 5.0 respectively, which are consistent with the levels observed in option market data (see, e.g., Broadie *et al.*, 2009). The returns on the put-delta-hedged and straddle portfolios are also in line with the values reported in previous empirical studies (Gonçalves and Guidolin, 2006; and Bernales and Guidolin, 2013).

The values of RV and IV should be equal to 5% in a Black-Sholes economy (where realized and implied volatilities are equal) since the dividend process volatility, σ, is set at 5% in our simulation setup. However, we can see in Table 1 that the agent's learning process generates an increase in RV and IV, although the rise in the level of IV is larger. This is explained by the effect of learning on the agent's beliefs. Learning induces that extreme events are perceived as more likely which also makes wider the risk-neutral probability distribution. This change in the shape of risk-neutral probability distribution strongly affects the IV, since it reflects the risk-neutralized volatility. Moreover, the learning effects on RV and IV are also influenced by the agent's attitude toward risks; the implied volatility (realized volatility) is 18%, 12% and 57% (6%, 6% and 8%) when α = 0.2, α = 0.5 and α = 5.0, respectively. Thus, learning produces a divergence between IV and RV which explains the 'existence' of the volatility risk premium.

Consistently with David and Veronesi (2002), Guidolin and Timmermann (2003) and Shaliastovich (2009), we report in Table 1 that learning induces an implied volatility surface which is reflected in the implied volatility slopes on the moneyness and maturity dimensions. Table 1 shows that the implied volatility slopes on the moneyness dimension ($Slope_{Mon}$) and on the maturity dimension ($Slope_{Mat}$) are different than zero for all values of the coefficient of relative risk aversion. For instance, the average value for $Slope_{Mon}$ (which is the simple difference between implied volatilities from short-term contracts with $K/S = 0.96$ and $K/S = 1.04$) is 0.05, 0.06 and -0.09 when α = 0.2, α = 0.5 and α = 5.0, respectively.

Table 1 reports that learning generates strong predictability patterns reflected in the first-order autocorrelations for the volatility risk premium, IV, and the implied volatility slopes (on moneyness and maturity dimensions). Immediately after a break, there is insufficient information, causing great uncertainty regarding the reliability of asset price estimations. This large uncertainty is reflected in a moderated increase in the realized volatility, but especially in an important growth in implied volatilities as explained above. Thus, there is a large volatility risk premium in the initial periods after a break which is accompanied by large implied volatility slopes on the moneyness and maturity dimensions (given the changes in the shape of the risk-neutral probability distribution). However, this uncertain reliability declines gradually as the agent learns over time. This learning process, where the agent uses new signals and historical information in a recursive updating procedure, induces predictive dynamics in the volatility risk premium, IV, and the implied volatility slopes. Similar predictability patterns have been also documented in the empirical literature. For instance, Cont and Fonseca (2002) and Gonçalves and Guidolin (2006) find that the implied volatility surfaces of S&P 500 options and FTSE 100 index options change dynamically over time, where the implied volatility and IVS slopes display high positive autocorrelations and mean reverting behaviours.

The average correlations of the variables of interest, generated through the model simulations, are reported in Table 2. Table 2 presents the results with a coefficient of relative risk aversion, α, at 0.2; however, we report additional correlation analyses in Appendix C, where we set α at 0.5 and 5.0.[19] Table 2 shows that the volatility risk premium is negatively related to option returns, which is consistent with Goyal and Saretto (2009) and Cao and Han (2013). In 49.32%, 93.71% and 82.48% of the simulations, the returns on put contracts, the put-delta-hedged portfolio and the straddle portfolio have negative significant correlations with the volatility risk premium, respectively. The intuition behind this result is straightforward. As discussed previously, when there is a break in the growth rate, there is an increase in market uncertainty due to learning which pushes up option prices and the volatility risk premium. Since option prices are larger after a break, option returns are smaller because the denominator of option returns is bigger (see Equation (12) and Equation (13)). Therefore, in our model when there is an elevated level of learning we have a high volatility risk premium and low option returns. In fact, option returns and the volatility risk premium are not only associated in the cross-section. In the following section, we will show, through a lagged regression analysis, that option returns and

[19] The results presented in Appendix C are congruent with the findings presented here

the volatility risk premium are also dynamically associated in the simulations of our learning model. Consequently, Table 2 presents the first set of evidence to support our contention that the agent's learning process may explain the relationship between option returns and the volatility risk premium.

[Insert Table 2 here]

Table 2 also presents evidence that learning can explain the negative relationship between risk-neutral skewness and option returns documented in previous empirical studies. For instance, Bali and Murray (2013) report that risk-neutral skewness is negatively related to option returns. Following Toft and Prucyk (1997), who show that the implied volatility slope of the IVS along the moneyness dimension ($Slope_{Mon}$) can be used as a proxy for risk-neutral skewness, Table 2 shows that there is an association generated by learning between option returns and risk-neutral skewness. For instance, in close to 42% of the simulations there are significantly negative correlations for the returns on put option contracts in relation to $Slope_{Mon}$. In the case of the returns of the put-delta-hedged portfolio and the straddle portfolio, the relationships with $Slope_{Mon}$ are also negative although only 22.28% and 13.95% of the simulations show significant correlations, respectively. Moreover, Table 2 reports evidence that the agent's learning process may help us to understand the association between option returns and the implied volatility term structure. Vasquez (2012) reports that the slope of the implied volatility term structure is positively related with option returns. In Table 2, $Slope_{Mat}$, which is the negative version of the measure used by Vasquez (2012), is negatively related to option returns. In 65.65% and 33.50% of the simulations, there are significantly negative correlations between $Slope_{Mat}$ and the returns on the put-delta-hedged portfolio and the straddle portfolio, respectively.

Furthermore, our results show that option returns are significantly correlated to the IV, which is also consistent with the empirical literature. For example, Gonçalves and Guidolin (2006) and Bernales and Guidolin (2013) report that abnormal economic profits, before transaction costs, are generated when the predictability patterns of implied volatilities are used to construct option portfolios. Table 2 shows that the implied volatility is significantly and negatively correlated with the returns on put option contracts, the put-delta-hedged portfolio and the straddle portfolio in 34.01%, 69.56%, and 31.97% of the simulations, respectively.

4 Predictability patterns of option returns

In order to examine whether learning can explain why the volatility risk premium predicts option returns, in this section, we firstly employ single-variable regression analyses accounting for several forecasting horizons. Afterwards, we consider single-variable and multivariate regressions to analyse the predictive power of the volatility risk premium and other option–implied variables, after including various control variables that have been used as traditional return predictors in financial markets. The forecasting exercise is based on linear regressions of the excess returns on put contracts, put-delta-hedged portfolios and straddle portfolios with different sets of lagged forecasting variables. The estimation is by OLS, and the t-statistics are computed using the Newey-West procedure to tackle the heteroscedasticity and serial correlations.

In Table 3 we begin by reporting the results for single-variable regressions of option returns with one- to twelve-month lagged volatility risk premium.[20] Note that in Table 3, the percentage of the simulations that have significant statistics is reported in parenthesis and is based on one-sided t-tests at the 5% significance level. Table 3 shows that 51.19% and 25.68% of the simulations obtain significant estimated coefficients of the one-month lagged volatility risk premium for the excess returns on put-delta-hedged portfolios (R_{DHput}) and straddle portfolios (R_{STRD}), respectively. This implies that the volatility risk premium is not only related to option returns in the cross-section (as reported in Table 2), but that the volatility risk premium also has predictive power for the returns on option contracts. The estimated coefficients of (IV-RV) are negative for R_{DHput} and R_{STRD}, implying that a high level of uncertainty proxied by the volatility risk premium makes more negative returns for option

[20] To save on space, we only present and discuss the simulation sample with α=0.2 in the main body of the paper. The regression results for the simulations with $\alpha = 0.5$ and $\alpha = 5.0$ are analogous to those with $\alpha = 0.2$, and are presented in Appendix C.

holders, which is congruent with empirical evidence documented by Goyal and Saretto (2009) and Cao and Han (2013).

It is may be noted in Table 3 that the returns on put-delta-hedged portfolios and straddle portfolios have a higher degree of predictability from the volatility risk premium relative to simple option put returns (i.e., only 12.41% of the simulations have significant estimated coefficients of the one-month lagged volatility risk premium for the excess returns on option put contracts). However, this is in line with the arguments in Broadie *et al*. (2009) and Bali and Murray (2013). Broadie *et al*. (2009) and Bali and Murray (2013) explain that since delta-hedged portfolios and straddle portfolios are free of risk caused by changes in the underlying asset price, they are more informative about potential deviations in option valuations than individual option contracts.

[Insert Table 3 here]

We also observe in Table 3 that the predictive power of the volatility risk premium is reduced as the forecasting horizons increase. The percentage of the simulations that have significant estimated coefficients declines as forecasting periods become longer. For instance, when twelve-month lagged *VRP* is used to predict R_{put}, R_{DHput} and R_{STRD}, only 7.99%, 14.97% and 12.07% of the simulations have a significant estimated coefficient, respectively. In addition, the average estimated coefficients of the one-month lagged *VRP* start at −2.69, −0.11 and −1.91 for R_{put}, R_{DHput} and R_{STRD}, respectively; nevertheless the average absolute value of the coefficients decreases gradually with larger forecasting horizons. The average estimated coefficients of the twelve-month lagged volatility risk premium for R_{put}, R_{DHput} and R_{STRD} is −0.04, −0.02 and −0.36, respectively. Figure 2 graphically presents the reduction in the predictability features of *VRP* regarding option returns when the forecasting periods increase. Taken as a whole, the results presented in Table 3 and Figure 2 reveal that the predictable patterns caused by the agent's learning process, in which the highest degree of predictability afforded by the volatility risk premium occurs at the one-month forecasting horizon, decline in longer forecasting horizons.

[Insert Figure 2 here]

Furthermore, Table 4 and Table 5 present a forecasting regression analysis in which we include additional predictor variables lagged in one-month and three-month periods, repetitively. Differently to Table 3, where the percentage of simulations that have significant statistics is based on one-sided t-tests, in Table 4 and Table 5 the same percentage values are based on two-sided t-tests since we do not want to impose any directional relationship in the multivariable regression analysis. At first glance, the results for put-delta-hedged returns and straddle returns reported in Table 4 Panel B and Panel C are most interesting since the predictability of (IV-RV) on option returns is strong, which is consistent to Table 3.

[Insert Table 4 here]

[Insert Table 5 here]

Table 4 reports that the implied volatility slopes on the moneyness and maturity dimensions, $Slope_{Mon}$ and $Slope_{Mat}$, exhibit a predictive power for option returns. For instance, the average estimated coefficient of $Slope_{Mon}$ and $Slope_{Mat}$ for R_{DHput} is -0.02 and -0.37, with 25.34% and 64.29% of the simulations obtaining statistically significant coefficients, respectively. These results are in line with Gonçalves and Guidolin (2006) and Bernales and Guidolin (2013), who find that the movements of the option IVS are highly predictable, leading to significant economic profits in portfolios with option contracts. Bali and Murray (2013) also report that risk-neutral skewness, which is associated with the implied volatility slope on the moneyness dimension, forecasts option returns. Moreover, Vasquez (2012) reports that the implied volatility term structure (i.e., the slope of IVS on the maturity dimension) has a forecasting power regarding returns on option contracts.

Likewise, Table 4 shows that IV has an important predictive power. The estimated coefficients of the one-month lagged IV are significant in 66.33% and 24.83% of the simulations for R_{DHput} and R_{STRD}, respectively. This is congruent with Gonçalves and Guidolin (2006), Jones (2006), Cao and Huang (2007) and Constantinides *et al.* (2013), who show that implied volatilities are dynamically associated to returns of option portfolios.

In relation to non-IV predictor variables in Table 4, although RV shows a negative relationship with option returns, which is consistent with the existing empirical literature (see Baski and Kapadia, 2003a,b; Bollerslev *et al*., 2009; Constantinides *et al*., 2013; Cao and Han, 2013), the predictive power that RV presents for the different option portfolio returns is much lower than predictive power of the volatility risk premium, IV and IVS slopes. For example, the single-variable regressions in Table 4 Panel B show that the average estimated coefficient of the one-month lagged RV for R_{DHput} is -0.03 and only 11.73% of the simulations have a significant coefficient. Conversely, the estimated coefficients of (IV-RV), IV, $Slope_{Mon}$ and $Slope_{Mat}$ obtain significant statistics in 43.37%, 66.33%, 25.34% and 64.29% of the simulations, respectively. In addition, Table 4 shows that other traditional predictor variables, such as the dividend yield, $DivYield$, and the excess return on the stock, $R_{m,t}$, also display much less predictive power over option returns.

Regarding the multivariate regressions reported in Table 4, we select predictor variables in such a way that multicollinearity problems are reduced. Table 4 shows that even after including other predictor variables such as $R_{m,t}$ and $DivYield$, the regressions involving (IV-RV) and other IV related predictor variables (i.e., IV, $Slope_{Mon}$ and $Slope_{Mat}$) still have an important predictability power on option returns. For instance, the multivariate regression in Table 4 Panel B shows with put-delta-hedged portfolio returns (R_{DHput}) that 42.18%, 65.99%, 25.34% and 63.78% of simulations have significant coefficients for the volatility risk premium, IV, $Slope_{Mon}$ and $Slope_{Mat}$, respectively.

The three-month forecasting regressions presented in Table 5 confirm the findings reported in Table 4. On the one side, the single-variable regressions in Table 5 have high percentage values of simulations with significant estimated coefficients for (IV-RV), IV and IVS slopes, especially for put-delta-hedged returns (R_{DHput}) and straddle returns (R_{STRD}). On the other side, the percentage of simulations with significant estimated coefficients is much lower for non-IV related predictors (i.e., RV, $DivYield$ and R_m) than for those option-implied variables. For instance, Table 5 Panel B shows that for put-delta-hedged returns (R_{DHput}), 29.93%, 48.64%, 18.20% and 46.94% of the simulations for (IV-RV), IV, $Slope_{Mon}$ and $Slope_{Mat}$ have significant estimated coefficients, respectively. Conversely, Table 5 Panel B reports that single-variable regressions for a maximum 10.88% of the simulations have significant estimated coefficients for non-IV related predictor variables. Moreover, the predictability features of option-implied variables on option returns are observable in multivariable regressions, even after adding other predictor variables such as $R_{m,t}$ and $DivYield$.

Nevertheless, the predictive power of the three-month lagged option-implied variables in Table 5 is lower than the predictive power reported with one-month lagged regressions in Table 4. The reduction in the predictive power is reflected in terms of the percentage of the simulations having significant statistics and adjusted R^2. This reinforces our findings in Table 3, where we show that the predictability features of the volatility risk premium decline with longer forecasting horizons.

5 Conclusions

In this paper, we consider a dynamic equilibrium model under rational learning to explain the puzzling predictive power of the volatility risk premium and other option-implied variables on option returns, which has been documented in the empirical literature. We extend the simple discrete-time endowment economy proposed by Lucas (1978), where we assume that the fundamental mean dividend growth rate is subject to breaks and is unknown by the representative agent. However, the agent learns recursively as new information arrives following a Bayesian updating procedure, and evaluates option prices accordingly.

Through an extensive set of simulations, we show that the learning process explains why the volatility risk premium can predict option returns. This explains the option return puzzle documented in Goyal and Saretto(2009) and Cao and Han (2013), where the volatility risk premium has a predictive power on returns of option portfolios. Our results are robust across various types of option trading strategies and the inclusion of alternative predictor variables.

Furthermore, we find that learning explains the forecasting features of the implied volatility and IVS slopes on returns of portfolios with option contracts. Thus, our learning model also provides an explanation for empirical studies regarding the relationship between predictive dynamics in implied volatilities and option returns (e.g., Gonçalves and Guidolin, 2006; Bernales and Guidolin, 2013), the forecasting relationship of option returns with implied skewness (e.g., Bali and Murray, 2013), and the association between the slope of the implied volatility term structure and predictions on option returns (e.g., Vasquez, 2012). Nevertheless, and similar to previous empirical studies, the predictive power induced by learning of the volatility risk premium, implied volatility and the implied volatility slopes is high in the one-month forecasting horizon, then tapers off over longer investment periods.

Finally, the model presented in our study is simple and intuitive, since it is based on generally accepted assumptions concerning preferences and the stochastic process of the fundamentals which drive asset prices. However, other interesting issues remain to be addressed. For instance, the study of cognitive mechanisms in a microstructure setup where trading is performed through market makers who also learn, the analysis of a model in which there are noisy and irrational traders, and the investigation of learning when there is asymmetric information among agents are left for future research.

Appendix A

Proof of Proposition II: Equation (4) and Equation (5) can be obtained from no-arbitrage arguments with respect to a contingent claim with values given by $\max\{K - S_{t+\tau}^{FI}\}$ and $\max\{S_{t+\tau}^{FI} - K\}$, respectively. Therefore, we have to prove that the probabilities that describe the state price density are risk-neutralized.

As a first step, we take the Euler equation of the stock price:

$$S_{t+k}^{FI} = \mathrm{E}_{t+k}\left[\beta\left(\frac{D_{t+\mathrm{k}+1}}{D_{t+k}}\right)^{-\alpha}(S_{t+k+1}^{FI} + D_{t+k+1})\right] \tag{A1}$$

Then, we divide both sides of Equation (A1) by the one-period zero-coupon bond with unit price to the expiration (obtained from Equation (3)):

$$\frac{(1+\rho)S_{t+k}^{FI}}{(1-\pi)(1+g_{t+k})^{-\alpha}+\pi\int_{g_d}^{g_u}(1+g_{t+k})^{-\alpha}dG(g_{t+k})} = \mathrm{E}_{t+k}\left[\beta\left(\frac{D_{t+\mathrm{k}+1}}{D_{t+k}}\right)^{-\alpha}\right.$$
$$\left.\cdot\,(S_{t+k+1}^{FI}+D_{t+k+1})\frac{(1+\rho)}{(1-\pi)(1+g_{t+k})^{-\alpha}+\pi\int_{g_d}^{g_u}(1+g_{t+k})^{-\alpha}dG(g_{t+k})}\right]. \tag{A2}$$

We know that the forward price and the forward cumulative dividend process are:

$$S_{t+k}^{FI^*} = \frac{(1+\rho)S_{t+k}^{FI}}{(1-\pi)(1+g_{t+k})^{-\alpha}+\pi\int_{g_d}^{g_u}(1+g_{t+k})^{-\alpha}dG(g_{t+k})} \tag{A3}$$

and

$$D_{t+k}^* = \sum_{\mathrm{s}=0}^{\mathrm{k}} D_{t+s}\frac{(1+\rho)}{(1-\pi)(1+g_{t+s})^{-\alpha}+\pi\int_{g_d}^{g_u}(1+g_{t+s})^{-\alpha}dG(g_{t+s})}. \tag{A4}$$

We also know from the pricing kernel that:

$$\mathrm{E}_t\left[\beta\left(\frac{D_{t+1}}{D_t}\right)^{-\alpha}\frac{(1+\rho)}{(1-\pi)(1+g_{t+k})^{-\alpha}+\pi\int_{g_d}^{g_u}(1+g_{t+k})^{-\alpha}dG(g_{t+k})}\right] = 1. \tag{A5}$$

If we add D_{t+k}^* to both sides of Equation (A2) and we use Equation (A5), then we obtain:

$$S_{t+k}^{FI^*} + D_{t+k}^*$$
$$= \mathrm{E}_{t+k}\left[\beta\left(\frac{D_{t+\mathrm{k}+1}}{D_{t+k}}\right)^{-\alpha}\frac{(1+\rho)}{(1-\pi)(1+g_{t+k})^{-\alpha}+\pi\int_{g_d}^{g_u}(1+g_{t+k})^{-\alpha}dG(g_{t+k})}(S_{t+k+1}^{FI^*}\right.$$
$$\left.+\,D_{t+k+1}^*)\right]. \tag{A6}$$

Consequently, through Equation (A6) we demonstrate that $S_{t+k}^{FI^*} + D_{t+k}^*$ follows a martingale under the conditional probability measure; and hence the risk-neutral density is:

$$\hat{p}_t\left(S_{t+k}^{FI}\right)$$
$$= \beta\left(\frac{D_{t+1}}{D_t}\right)^{-\alpha}\frac{(1+\rho)}{(1-\pi)(1+g_{t+k})^{-\alpha}+\pi\int_{g_d}^{g_u}(1+g_{t+k})^{-\alpha}dG(g_{t+k})}p_t(D_{t+k}). \tag{A7}$$

Thus, the one-period state-price density is:

$$\tilde{p}_t\left(S_{t+k}^{FI}\right) = \beta\left(\frac{D_{t+\mathrm{k}}}{D_t}\right)^{-\alpha}\frac{1}{1+r_{t+k}^{FI}}$$
$$\cdot\frac{(1+\rho)}{(1-\pi)(1+g_{t+k})^{-\alpha}+\pi\int_{g_d}^{g_u}(1+g_{t+k})^{-\alpha}dG(g_{t+k})}p_t(D_{t+k}) \qquad \text{(A8)}$$
$$= \beta\left(\frac{D_{t+1}}{D_t}\right)^{-\alpha}p_t(D_{t+k})$$

where r_{t+k}^{FI} is the one-period risk-free interest rate.

Moreover, if the risk-neutral measure on a single-period model is unique and exists, this is a sufficient condition to have a unique risk-neutral measure on an infinite-period economy obtained by repetition of several single-period models [see Pliska (1997) for proof]. In our model, the infinite-period model risk-neutral measure can be characterized by using the independence of breaks on the mean dividend growth rates and taking all paths that could guide to a particular state in $t+\tau$ periods ahead. Thus, $\tilde{p}_t(S_{t+\tau}^{FI})$ is the state price density of all paths that lead to the state in which the dividend is $D_{t+\tau}$, where the expected value of $D_{t+\tau}$ is:

$$\mathrm{E_t}[D_{t+\tau}] = D_t\mathrm{E}_t\left[\frac{D_{t+1}}{D_t}\mathrm{E}_{t+1}\left[\left(\frac{D_{t+2}}{D_{t+1}}\right)\dots\mathrm{E}_{t+\tau-1}\left[\left(\frac{D_{t+\tau}}{D_{t+\tau-1}}\right)\right]\right]\right]. \qquad \text{(A9)}$$

In addition, using the independence of $\{\varepsilon_{t+i}\}_{i=1}^{\tau}$ and $\{g_{t+i-1}\}_{i=1}^{\tau}$ we have:

$$\mathrm{E_t}[D_{t+\tau}] = D_t\mathrm{E_t}\left[\exp\left(\sqrt{\tau}\sigma\varepsilon_{t+\tau}-\tau\sigma^2/2\right)\prod_{i=1}^{\tau}(1+g_{t+i-1})\right]. \qquad \text{(A10)}$$

Furthermore, let z be the number of breaks between t and $t+\tau$ that is a random variable drawn from a Binomial distribution, $\varphi(z|\tau,\pi)$, with parameters τ and π; while $\{h_i\}_{i=1}^{z}$ are the time periods between breaks which are also random variables that follow a geometric distribution with parameter π, $\eta(h_i|\pi)$, where $\tau=\sum_{i=1}^{z}h_i$. Therefore, in each path we have:

$$D_{t+\tau}^{FI} = D_t\exp\left(\sqrt{\tau}\sigma\varepsilon_{t+\tau}-\tau\sigma^2/2\right)\prod_{i=1}^{z}(1+g_{t+h_i})^{h_i}. \qquad \text{(A11)}$$

Here $\{g_{t+h_i}\}_{i=2}^{z}$ are drawn from a univariate density $g_{t+h_i}\sim G(\cdot)$ and pdf $\varrho(g_{t+h_i})$ defined on the support $[g_d, g_u]$; while $g_{t+h_1}=g_t$ and $g_{t+\tau}=g_{t+h_z}$. Consequently,

$$p_t(D_{t+\tau}) = \phi(\varepsilon_{t+\tau}|0,\sigma)\varphi(z|\tau,\pi)\eta(h_1|\pi)\left[\eta(h_2|\pi)\varrho\left(g_{t+h_2}\right) \dots \eta(h_z|\pi)\varrho\left(g_{t+h_z}\right)\right] \quad . \qquad \text{(A12)}$$

Hence from Equation (A8) we can write:

$$\tilde{p}_t(S^{FI}_{t+\tau}) \\ = \beta^\tau \left(\frac{D_{t+\tau}}{D_t}\right)^{-\alpha} \phi(\varepsilon_{t+\tau}|0,\sigma)\varphi(z|\tau,\pi)\eta(h_1|\pi)\left[\eta(h_2|\pi)\varrho\left(g_{t+h_2}\right) \dots \eta(h_z|\pi)\varrho\left(g_{t+h_z}\right)\right]. \qquad \text{(A13)}$$

□

Appendix B

Probability of breaks in the mean dividend growth rate

We estimate the probability of breaks in the mean dividend growth rate through the test introduced by Chu *et al.* (1996). Chu *et al.* (1996) introduce a dynamic test with which we can contemporaneously detect a break on a given date due to the real-time features of their algorithm. As in our model, let us assume that real dividends evolve following a geometric random walk: $\ln(D_{t+1}/D_t) = \mu_{t+1} + \sigma\varepsilon_{t+1}$. Let v be the minimum number of periods over which the drift, μ_{t+1}, is assumed to be constant, given that n is the number of periods from the last break (i.e. $\mu_{t-n+1} = \mu_{t-n+2} = \ldots = \mu_{t-n+v}$). Therefore, the representative agent starts detecting the presence of breaks after the span period v. In this context, Chu *et al.* (1996) propose the use of the following fluctuation detector in the case of a univariate location model:

$$\hat{Z}_t = n\hat{s}_0^{-1}(\hat{\mu}_t - \hat{\mu}_v). \qquad \text{(B-1)}$$

Here, $\hat{\mu}_t$ and $\hat{\mu}_v$ are the parameter estimates at time t and v. We defined $\boldsymbol{\xi}_t$ as the vector of signals about μ_t in Equation (6) and Equation (7); therefore $\hat{\mu}_t = \bar{\xi}_t = (1/n)\sum_{i=t-n+1}^{t}\xi_i$, $\hat{\mu}_v = \bar{\xi}_v = \left(\frac{1}{v}\right)\sum_{i=t-n+1}^{t-n+v}\xi_i$ and $\hat{s}_0 = (v^{-1}\sum_{i=t-n+1}^{t-n+v}(\xi_i - \hat{\mu}_v)^2)^{0.5}$. Assuming a null hypothesis of no break, Chu *et al.* (1996) present asymptotic bounds for the statistic $|\hat{Z}_t|$:

$$\lim_{v\to\infty} P\left\{|\hat{Z}_t| \geq \sqrt{v}\left(\frac{n-v}{v}\right)\left[\left(\frac{n}{n-v}\right)\left[a^2 + \ln\left(\frac{n}{n-v}\right)\right]\right]^{\frac{1}{2}}\right\} \cong 2\left(1 - \Phi(a) + a\phi(a)\right), \qquad \text{(B-2)}$$

where $\Phi(\cdot)$ and $\phi(\cdot)$ are the cdf and pdf of a standard normal random variable, respectively, while a is a constant related to the significance level of the test. Therefore, the intuition behind this test is that given a significance level, an agent could start the calculation of $\hat{Z}_t$ recursively and in real-time after v signals received from the previous break, with the objective of detecting a new one. The testing process starts again after the detection of each new break. In this paper we assume that dividends are paid out daily, which is true for wide market indexes. For that reason, we use daily dividend time series from the S&P 500 index between 1996 and 2007, which were deseasonalized and adjusted by the consumer price index. We set v=125 which represents six months of trading dates, and we use 5% significance. We detect eight breaks in the period between 1996 and 2007. Figure B-1 shows the breaks detected in this period.

[Insert Figure B-1 here]

Appendix C

Additional experiments and robustness checks

This appendix presents simulation results for additional parameter setups. Table C-1 and Table C-2 show correlation analyses with a coefficient of relative risk aversion equal to 0.5 and 5.0, respectively. Table C-3 and Table C-4 report volatility risk premium regressions also with a coefficient of relative risk aversion equal to 0.5 and 5.0, respectively. Finally, Table C-5 and Table C-6 (Table C-7 and Table C-8) present single-variable and multivariable regressions of one-month lagged (three-month lagged) predictor variables in which the level of relative risk aversion is set at 0.5 and 5.0, respectively.

It is important to notice that when $\alpha > 1$, in general representative agent endowment-based asset pricing models display a counter-intuitive feature by which stock prices are lower when g_t increases (see Abel, 1988; Cecchetti *et al.*, 1990). Despite this counter-intuitive properties of dynamic equilibrium models when $\alpha > 1$, we include them in our analyses to be consistent with the large literature in which α > 1 has been estimated or used to explain properties of asset prices.

[Insert Table C-1 here]

[Insert Table C-2 here]

[Insert Table C-3 here]

[Insert Table C-4 here]

[Insert Table C-5 here]

[Insert Table C-6 here]

[Insert Table C-7 here]

[Insert Table C-8 here]

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Table 1
Summary statistics

The table contains summary statistics of the main variables in the model simulations performed in our study. The simulations are based on an economy with breaks and incomplete information under Bayesian learning. The table reports results for three coefficients of relative risk aversion ($\alpha = 0.2$, $\alpha = 0.5$ and $\alpha = 5.0$). $R_{put,t}$ is the excess return on an at-the-money one-month to maturity put option contract, $R_{DHput,t}$ is the excess return on a put-delta-hedged portfolio that is formed by buying one at-the-money one-month to maturity put option contract and buying delta shares of the stock, $R_{STRD,t}$ is the excess return on a straddle portfolio that is calculated as a combination of one call option contract and one put option contract; where both contracts are at-the-money one-month to maturity. $R_{put,t}$, $R_{DHput,t}$ and $R_{STRD,t}$ are obtained using a hold-to-expiration trading strategy on a monthly basis and with non-overlapping intervals as in Ni (2009) and Broadie *et al*. (2009). The implied volatility, IV_t, is expressed on an annual basis and calculated as the average of the Black-Scholes' (1973) implied volatilities of a European put option contract and a European call option contract, where both contracts are at-the-money and with one-month to maturity; while RV_t is the standard deviation (annual basis) of the daily stock log-returns in each month also avoiding overlapping periods. The slope on the moneyness dimension of the implied volatility surface, $Slope_{Mon}$, is calculated as the difference between the implied volatility of contracts with $K/S = 0.96$ and one-month to maturity and the implied volatility of contracts with $K/S = 1.04$ and one-month to maturity. The slope on the maturity dimension of the implied volatility surface, $Slope_{Mat}$, is calculated as the difference between the implied volatility of at-the-money contracts with one-month to maturity and the implied volatility of at-the-money contracts with three-months to maturity. The AR(1) statistics are the values of the LM test for ARCH effects suggested by Engle (1982) using one lag. The numbers in the table are the average estimates over 2,000 simulations; for each of these simulations, we generate 12 years (3,024 days) of daily dividends. The percentage of the simulations with significant statistics for the respective diagnostic tests is reported in parentheses at 5% significance.

Scenario		$R_{put,t}$	$R_{DHput,t}$	$R_{STRD,t}$	IV_t - RV_t	IV_t	RV_t	$Slope_{Mon,}$	$Slope_{Mat,t}$
					$\alpha = 0.2$				
	Mean	-0.55	-0.02	-0.50	0.12	0.18	0.06	0.05	0.04
	Std. dev.	0.76	0.01	0.39	0.02	0.02	0.01	0.06	0.01
	Skewness	2.01	0.89	1.14	-0.28	-0.39	1.17	-0.11	-0.61
	Kurtosis	7.84	4.12	4.80	5.67	6.63	7.75	4.36	7.49
	AR(1)	-0.01	0.03	0.00	0.40	0.82	0.06	0.77	0.80
		(4.76)	(11.05)	(6.46)	(89.80)	(99.32)	(14.12)	(99.66)	(98.81)
					$\alpha = 0.5$				
	Mean	-0.39	-0.01	-0.35	0.06	0.12	0.06	0.06	0.02
	Std. dev.	1.02	0.01	0.51	0.01	0.01	0.01	0.05	0.01
Breaks - Inc. Inf. (Learning)	Skewness	1.96	0.92	1.02	-0.09	-0.24	0.48	-0.37	-0.37
	Kurtosis	7.17	4.05	4.00	4.80	7.03	4.17	5.01	7.92
	AR(1)	-0.01	0.00	0.00	0.24	0.80	0.03	0.71	0.75
		(4.59)	(7.48)	(6.63)	(66.84)	(98.98)	(8.16)	(99.49)	(98.47)
					$\alpha = 5.0$				
	Mean	-0.96	-0.08	-0.92	0.48	0.57	0.08	-0.09	0.17
	Std. dev.	0.30	0.02	0.28	0.09	0.09	0.05	0.10	0.04
	Skewness	7.41	2.20	7.05	-0.58	-0.14	6.03	-0.68	0.03
	Kurtosis	64.96	14.04	60.35	5.56	4.18	43.96	3.76	5.49
	AR(1)	0.01	0.19	0.00	0.71	0.84	0.03	0.87	0.81
		(3.83)	(51.11)	(1.05)	(98.08)	(99.83)	(2.79)	(100.00)	(99.65)

Table 2
Correlation matrix

The table reports a correlation analysis of the main variables generated in the model simulations performed in our study. The simulations are based on an economy with breaks and incomplete information under Bayesian learning. This table presents the results with a coefficient of relative risk aversion, α, at 0.2; however, additional correlation analyses with different coefficients of risk aversion are reported in Appendix C. The variables $R_{put,t}$, $R_{DHput,t}$, $R_{STRD,t}$, IV_t, RV_t, $Slope_{Mon,t}$ $Slope_{Mat,t}$ are defined in Table 1. The numbers in the table are the average estimates over 2,000 simulations; for each of these simulations, we generate 12 years (3,024 days) of daily dividends. The percentage of the simulations with significant statistics for the respective diagnostic tests is reported in parentheses at 5% significance.

	$R_{put,t}$	$R_{DHput,t}$	$R_{STRD,t}$	IV_t - RV_t	IV_t	RV_t	$Slope_{Mon,t}$	$Slope_{Mat,t}$
	Breaks - Inc. Inf. (Learning) and α = 0.2							
$R_{put,t}$	1.00	0.49	0.37	-0.16	-0.09	0.13	-0.14	-0.05
	(100.00)	(100.00)	(98.64)	(49.32)	(34.01)	(33.84)	(41.50)	(29.42)
$R_{DHput,t}$		1.00	0.97	-0.32	-0.22	0.21	-0.06	-0.21
		(100.00)	(100.00)	(93.71)	(69.56)	(63.61)	(22.28)	(65.65)
$R_{STRD,t}$			1.00	-0.26	-0.12	0.23	-0.02	-0.12
			(100.00)	(82.48)	(31.97)	(70.75)	(13.95)	(33.50)
IV_t - RV_t				1.00	0.65	-0.65	0.35	0.59
				(100.00)	(100.00)	(97.96)	(84.52)	(99.83)
IV_t					1.00	0.12	0.22	0.95
					(100.00)	(42.35)	(71.60)	(100.00)
RV_t						1.00	-0.27	0.16
						(100.00)	(81.29)	(51.53)
$Slope_{Mon,t}$							1.00	0.07
							(100.00)	(64.63)
$Slope_{Mat,t}$								1.00
								(100.00)

Table 3
Volatility risk premium regressions
The table presents single-variable regressions of lagged the volatility risk premium on hold-to-maturity put option returns (R_{put}), put-delta-hedged portfolio returns (R_{DHput}) and straddle portfolio returns (R_{STRD}) over 1-, 3-, 6-, 9- and 12-month forecasting horizons. The variables ($IV_t - RV_t$), $R_{put,t}$, $R_{DHput,t}$, $R_{STRD,t}$ are defined in Table 1. The simulations are based on an economy with breaks and incomplete information under Bayesian learning. This table presents the results with a coefficient of relative risk aversion, α, at 0.2; however, additional regression analyses with different coefficients of risk aversion are reported in Appendix C. To adjust for heteroscedasticity and serial correlation, robust Newey-West (1987) t-statistics are used in the t-tests. The numbers in the table are the average estimates over 2,000 simulations; for each of these simulations, we generate 12 years (3,024 days) of daily dividends. The percentage of the simulations with significant statistics for the respective diagnostic tests is reported in parentheses at 5% significance on one-sided t-tests.

Monthly forecasting horizons	1	3	6	9	12
	Breaks - Inc. Inf. (Learning) and α = 0.2				
	Dependent Variable $R_{put,t}$				
Constant	-0.37	-0.48	-0.50	-0.51	-0.54
	(41.50)	(55.61)	(58.67)	(60.03)	(64.80)
IV_t - RV_t	-2.69	-1.03	-0.64	-0.49	-0.04
	(12.41)	(8.67)	(9.18)	(7.14)	(7.99)
Adj. R^2 (%)	0.55	0.04	0.06	-0.08	0.04
	Dependent Variable $R_{DHput,t}$				
Constant	-0.01	-0.01	-0.01	-0.01	-0.01
	(56.29)	(67.18)	(77.55)	(85.20)	(87.07)
IV_t - RV_t	-0.11	-0.09	-0.06	-0.04	-0.02
	(51.19)	(38.10)	(27.04)	(16.67)	(14.97)
Adj. R^2 (%)	3.34	1.87	1.05	0.59	0.35
	Dependent Variable $R_{STRD,t}$				
Constant	-0.37	-0.40	-0.43	-0.46	-0.48
	(75.85)	(78.74)	(84.18)	(87.93)	(89.97)
IV_t - RV_t	-1.91	-1.56	-1.02	-0.66	-0.36
	(25.68)	(22.62)	(14.63)	(11.05)	(12.07)
Adj. R^2 (%)	0.92	0.64	0.31	0.17	0.06

Table 4
Monthly return regression
The table reports single-variable and multivariable regressions of one-month lagged predictor variables on hold-to-maturity put option contract returns R_{put} (Panel A), put-delta-hedged portfolio returns R_{DHput} (Panel B) and straddle portfolio returns R_{STRD} (Panel C). The simulations are based on an economy with breaks and incomplete information under Bayesian learning. This table presents the results with a coefficient of relative risk aversion, α, at 0.2; however, additional regression analyses with different coefficients of risk aversion are reported in Appendix C. The variables $(IV_t - RV_t)$, $R_{put,t}$, $R_{DHput,t}$, $R_{STRD,t}$, IV_t, RV_t, $Slope_{Mon,t}$ $Slope_{Mat,t}$ are defined in Table 1. The dividend yield, $DivYield_t$, is implicitly obtained from the call-put parity relationship of the European put and call option contracts (at-the-money one-month to maturity contracts). $R_{m,t}$ is the excess return on the stock (the stock price is calculated with Equation (8)). To adjust for heteroscedasticity and serial correlation, robust Newey-West (1987) t-statistics are used in the t-tests. The numbers in the table are the average estimates over 2,000 simulations; for each of these simulations, we generate 12 years (3,024 days) of daily dividends. The percentage of the simulations with significant statistics for the respective diagnostic tests is reported in parentheses at 5% significance on two-sided t-tests.

Panel A.	Monthly return regression Breaks - Inc. Inf. (Learning) and α = 0.2											
	Dependent Variable $R_{put.t}$											
Constant	-0.37	0.16	-0.52	-0.53	-0.46	-0.35	-0.55	-0.01	0.54	-0.27	-0.11	-0.21
	(32.65)	(6.63)	(34.18)	(97.28)	(28.91)	(7.48)	(99.66)	(8.33)	(6.46)	(7.99)	(9.18)	(8.84)
IV_t - RV_t	-2.69							-2.68				
	(7.99)							(7.99)				
IV_t		-5.30							-5.43			
		(9.01)							(9.52)			
RV_t			-0.44							-0.41		
			(6.12)							(6.12)		
$Slope_{Mon.t}$				-0.37							-0.45	
				(6.46)							(8.16)	
$Slope_{Mat.t}$					-2.32							-2.23
					(6.63)							(6.63)
$DivYield_t$						-3.50		-6.13	-6.21	-4.40	-7.15	-4.37
						(4.93)		(7.14)	(7.48)	(7.31)	(7.99)	(8.16)
$R_{m.t}$							0.31	0.56	0.50	0.42	0.78	0.31
							(5.78)	(5.95)	(5.95)	(6.29)	(6.12)	(5.95)
Adj. R^2 (%)	0.55	0.99	0.02	0.12	0.34	0.02	-0.04	0.57	1.00	0.04	0.20	0.34

Panel B.	Monthly return regression Breaks - Inc. Inf. (Learning) and α = 0.2											
	Dependent Variable $R_{DHput.t}$											
Constant	-0.01	0.01	-0.01	-0.02	0.00	-0.02	-0.02	-0.01	0.01	-0.02	-0.02	-0.01
	(44.73)	(25.51)	(68.37)	(99.83)	(14.29)	(21.60)	(100.00)	(12.59)	(7.99)	(18.88)	(17.01)	(10.03)
IV_t - RV_t	-0.11							-0.11				
	(43.37)							(42.18)				
IV_t		-0.22							-0.22			
		(66.33)							(65.99)			
RV_t			-0.03							-0.02		
			(11.73)							(11.56)		
$Slope_{Mon.t}$				-0.02							-0.02	
				(25.34)							(25.34)	
$Slope_{Mat.t}$					-0.37							-0.37
					(64.29)							(63.78)
$DivYield_t$						0.13		0.10	0.07	0.15	0.10	0.12
						(12.76)		(10.54)	(8.84)	(14.46)	(12.41)	(10.20)
$R_{m.t}$							0.00	-0.01	0.00	-0.01	0.00	-0.01
							(6.80)	(6.80)	(6.29)	(8.67)	(7.14)	(7.31)
Adj. R^2 (%)	3.34	5.85	0.32	1.93	5.45	0.24	0.12	3.43	5.85	0.68	2.12	5.50

Panel C.	Monthly return regression Breaks - Inc. Inf. (Learning) and α = 0.2											
	Dependent Variable $R_{STRD.t}$											
Constant	-0.37	-0.01	-0.47	-0.48	-0.22	-0.72	-0.50	-0.56	-0.16	-0.72	-0.67	-0.42
	(65.65)	(9.35)	(72.79)	(100.00)	(33.16)	(17.86)	(100.00)	(13.61)	(8.50)	(16.67)	(16.50)	(12.24)
IV_t - RV_t	-1.91							-1.84				
	(17.86)							(17.35)				
IV_t		-3.75							-3.74			
		(24.83)							(23.81)			
RV_t			-0.51							-0.44		
			(9.01)							(8.67)		
$Slope_{Mon.t}$				-0.33							-0.31	
				(14.46)							(13.78)	
$Slope_{Mat.t}$					-6.90							-6.93
					(25.34)							(25.17)
$DivYield_t$						3.77		3.17	2.57	4.27	3.08	3.40
						(10.03)		(8.67)	(7.31)	(9.69)	(9.01)	(8.16)
$R_{m.t}$							-0.08	-0.11	-0.03	-0.18	-0.07	-0.14
							(6.97)	(6.97)	(6.46)	(7.48)	(6.63)	(7.14)
Adj. R^2 (%)	0.92	1.87	0.10	0.48	1.83	0.09	0.03	0.97	1.88	0.23	0.56	1.86

Table 5
Three-month return regressions
The table reports single-variable and multivariable regressions of three-month lagged predictor variables on hold-to-maturity put option contract returns R_{put} (Panel A), put-delta-hedged portfolio returns R_{DHput} (Panel B) and straddle portfolio returns R_{STRD} (Panel C). The simulations are based on an economy with breaks and incomplete information under Bayesian learning. This table presents the results with a coefficient of relative risk aversion, α, at 0.2; however, additional regression analyses with different coefficients of risk aversion are reported in Appendix C. The variables $(IV_t - RV_t)$, $R_{put,t}$, $R_{DHput,t}$, $R_{STRD,t}$, IV_t, RV_t, $Slope_{Mon,t}$ $Slope_{Mat,t}$ are defined in Table 1. The dividend yield, $DivYield_t$ and $R_{m,t}$ are defined in Table 4. To adjust for heteroscedasticity and serial correlation, robust Newey-West (1987) t-statistics are used in the t-tests. The numbers in the table are the average estimates over 2,000 simulations; for each of these simulations, we generate 12 years (3,024 days) of daily dividends. The percentage of the simulations with significant statistics for the respective diagnostic tests is reported in parentheses at 5% significance on two-sided t-tests.

Panel A.	Three-month return regression Breaks - Inc. Inf. (Learning) and α = 0.2											
	Dependent Variable $R_{put,t}$											
Constant	-0.48	-0.25	-0.51	-0.56	-0.42	-0.24	-0.55	-0.07	0.18	-0.15	-0.20	-0.07
	(44.56)	(11.73)	(35.37)	(98.64)	(27.89)	(7.31)	(99.83)	(5.78)	(5.27)	(6.12)	(6.29)	(5.61)
IV_t - RV_t	-1.03							-1.04				
	(6.80)							(6.29)				
IV_t		-2.28							-2.38			
		(9.52)							(9.01)			
RV_t			-0.60							-0.57		
			(6.80)							(7.14)		
$Slope_{Mon.t}$				0.14							0.10	
				(7.14)							(6.46)	
$Slope_{Mat,t}$					-3.24							-3.30
					(8.33)							(8.16)
$DivYield_t$						-5.29		-6.93	-7.06	-6.22	-6.12	-5.98
						(5.95)		(5.27)	(5.61)	(5.61)	(5.61)	(5.61)
$R_{m,t}$							0.11	0.35	0.31	0.32	0.31	0.20
							(6.29)	(4.08)	(4.59)	(3.74)	(4.59)	(4.59)
Adj. R^2 (%)	0.04	0.23	-0.04	0.01	0.19	0.03	-0.02	0.01	0.21	-0.07	0.00	0.16

Panel B.	Three-month return regression Breaks - Inc. Inf. (Learning) and α = 0.2											
	Dependent Variable $R_{DHput,t}$											
Constant	-0.01	0.00	-0.02	-0.02	-0.01	-0.01	-0.02	-0.01	0.01	-0.01	-0.01	0.00
	(55.95)	(15.99)	(73.13)	(99.83)	(28.23)	(14.63)	(100.00)	(8.50)	(10.37)	(14.80)	(12.07)	(9.69)
IV_t - RV_t	-0.09							-0.08				
	(29.93)							(27.72)				
IV_t		-0.16							-0.16			
		(48.64)							(46.77)			
RV_t			-0.01							-0.01		
			(10.88)							(9.18)		
$Slope_{Mon,t}$				-0.01							-0.01	
				(18.20)							(17.69)	
$Slope_{Mat,t}$					-0.25							-0.25
					(46.94)							(44.39)
$DivYield_t$						-0.03		-0.08	-0.11	-0.03	-0.07	-0.07
						(10.88)		(8.50)	(10.37)	(10.20)	(9.69)	(9.35)
$R_{m,t}$							0.00	0.00	0.00	0.00	0.00	0.00
							(6.46)	(5.95)	(5.61)	(5.78)	(5.44)	(5.61)
Adj. R^2 (%)	1.87	3.18	0.18	1.03	2.86	0.24	0.00	1.96	3.22	0.40	1.19	2.92

Panel C.	Three-month return regression Breaks - Inc. Inf. (Learning) and α = 0.2											
	Dependent Variable $R_{STRD,t}$											
Constant	-0.40	-0.15	-0.49	-0.49	-0.33	-0.48	-0.50	-0.33	-0.04	-0.48	-0.42	-0.25
	(69.56)	(13.10)	(75.51)	(100.00)	(42.69)	(13.95)	(100.00)	(10.20)	(10.03)	(12.93)	(12.76)	(10.88)
IV_t - RV_t	-1.56							-1.52				
	(15.99)							(15.65)				
IV_t		-2.67							-2.67			
		(21.60)							(21.26)			
RV_t			-0.15							-0.09		
			(8.67)							(7.82)		
$Slope_{Mon,t}$				-0.24							-0.25	
				(12.59)							(11.39)	
$Slope_{Mat,t}$					-4.37							-4.37
					(21.43)							(21.77)
$DivYield_t$						-0.37		-1.23	-1.96	-0.27	-1.13	-1.39
						(7.65)		(8.16)	(8.67)	(8.16)	(8.50)	(9.35)
$R_{m,t}$							-0.08	-0.03	0.06	-0.06	0.00	-0.02
							(6.29)	(5.78)	(5.78)	(5.78)	(4.76)	(5.27)
Adj. R^2 (%)	0.64	1.09	0.08	0.35	1.03	0.13	-0.05	0.67	1.11	0.13	0.37	1.06

Table C-1
Correlation matrix with a coefficient of relative risk aversion equal to 0.5
The table reports a correlation analysis of the main variables generated in the model simulations performed in our study. The simulations are based on an economy with breaks and incomplete information under Bayesian learning. This table presents the results with a coefficient of relative risk aversion, α, at 0.5; however, additional correlation analyses with different coefficients of risk aversion are reported in Appendix C. The variables $R_{put,t}$, $R_{DHput,t}$, $R_{STRD,t}$, IV_t, RV_t, $Slope_{Mon,t}$ $Slope_{Mat,t}$ are defined in Table 1. The numbers in the table are the average estimates over 2,000 simulations; for each of these simulations, we generate 12 years (3,024 days) of daily dividends. The percentage of the simulations with significant statistics for the respective diagnostic tests is reported in parentheses at 5% significance.

	$R_{put,t}$	$R_{DHput,t}$	$R_{STRD,t}$	IV_t - RV_t	IV_t	RV_t	$Slope_{Mon,t}$	$Slope_{Mat,t}$
	Breaks - Inc. Inf. (Learning) and α = 0.5							
$R_{put,t}$	1.00	0.53	0.36	-0.14	-0.07	0.11	-0.08	-0.05
	(100.00)	(100.00)	(96.94)	(38.10)	(29.08)	(26.87)	(17.86)	(25.00)
$R_{DHput,t}$		1.00	0.97	-0.25	-0.15	0.19	-0.03	-0.14
		(100.00)	(100.00)	(82.14)	(43.88)	(61.22)	(14.46)	(39.29)
$R_{STRD,t}$			1.00	-0.23	-0.11	0.19	0.00	-0.11
			(100.00)	(75.34)	(28.40)	(62.24)	(10.37)	(29.08)
IV_t - RV_t				1.00	0.52	-0.79	0.32	0.47
				(100.00)	(100.00)	(99.66)	(82.14)	(98.81)
IV_t					1.00	0.08	0.31	0.92
					(100.00)	(30.78)	(77.89)	(100.00)
RV_t						1.00	-0.17	0.09
						(100.00)	(51.87)	(31.80)
$Slope_{Mon,t}$							1.00	0.22
							(100.00)	(68.88)
$Slope_{Mat,t}$								1.00
								(100.00)

Table C-2
Correlation matrix with a coefficient of relative risk aversion equal to 5.0
The table reports a correlation analysis of the main variables generated in the model simulations performed in our study. The simulations are based on an economy with breaks and incomplete information under Bayesian learning. This table presents the results with a coefficient of relative risk aversion, α, at 5.0; however, additional correlation analyses with different coefficients of risk aversion are reported in Appendix C. The variables $R_{put,t}$, $R_{DHput,t}$, $R_{STRD,t}$, IV_t, RV_t, $Slope_{Mon,t}$ $Slope_{Mat,t}$ are defined in Table 1. The numbers in the table are the average estimates over 2,000 simulations; for each of these simulations, we generate 12 years (3,024 days) of daily dividends. The percentage of the simulations with significant statistics for the respective diagnostic tests is reported in parentheses at 5% significance.

	$R_{put,t}$	$R_{DHput,t}$	$R_{STRD,t}$	IV_t - RV_t	IV_t	RV_t	$Slope_{Mon,t}$	$Slope_{Mat,t}$
	Breaks - Inc. Inf. (Learning) and α = 5.0							
$R_{put,t}$	1.00	0.35	0.47	-0.19	0.15	0.56	0.11	0.16
	(100.00)	(81.18)	(77.87)	(58.89)	(58.71)	(90.07)	(14.98)	(58.19)
$R_{DHput,t}$		1.00	0.72	-0.82	-0.45	0.59	-0.21	-0.41
		(100.00)	(100.00)	(99.83)	(93.73)	(96.52)	(73.17)	(91.64)
$R_{STRD,t}$			1.00	-0.38	0.18	0.92	0.12	0.19
			(100.00)	(91.29)	(57.84)	(100.00)	(18.64)	(64.46)
IV_t - RV_t				1.00	0.80	-0.28	0.50	0.74
				(100.00)	(100.00)	(75.44)	(91.81)	(99.83)
IV_t					1.00	0.31	0.60	0.95
					(100.00)	(92.51)	(93.03)	(99.83)
RV_t						1.00	0.20	0.32
						(100.00)	(62.20)	(90.24)
$Slope_{Mon,t}$							1.00	0.50
							(100.00)	(90.07)
$Slope_{Mat,t}$								1.00
								(100.00)

Table C-3

Volatility risk premium regressions with a coefficient of relative risk aversion equal to 0.5

The table presents single-variable regressions of lagged the volatility risk premium on hold-to-maturity put option returns (R_{put}), put-delta-hedged portfolio returns (R_{DHput}) and straddle portfolio returns (R_{STRD}) over 1-, 3-, 6-, 9- and 12-month forecasting horizons. The variables ($IV_t - RV_t$), $R_{put,t}$, $R_{DHput,t}$, $R_{STRD,t}$ are defined in Table 1. The simulations are based on an economy with breaks and incomplete information under Bayesian learning. This table presents the results with a coefficient of relative risk aversion, α, at 0.5; however, additional regression analyses with different coefficients of risk aversion are reported in Appendix C. To adjust for heteroscedasticity and serial correlation, robust Newey-West (1987) t-statistics are used in the t-tests. The numbers in the table are the average estimates over 2,000 simulations; for each of these simulations, we generate 12 years (3,024 days) of daily dividends. The percentage of the simulations with significant statistics for the respective diagnostic tests is reported in parentheses at 5% significance on one-sided t-tests.

Monthly forecasting horizons	1	3	6	9	12
	Breaks - Inc. Inf. (Learning) and α = 0.5				
	Dependent Variable $R_{put,t}$				
Constant	-0.26	-0.36	-0.37	-0.37	-0.39
	(31.97)	(40.82)	(43.37)	(42.18)	(48.98)
IV_t - RV_t	-3.93	-0.86	-0.54	-0.32	0.25
	(10.71)	(6.80)	(8.16)	(6.80)	(8.67)
Adj. R^2 (%)	0.38	-0.04	0.04	-0.10	0.06
	Dependent Variable $R_{DHput,t}$				
Constant	-0.01	-0.01	-0.01	-0.01	-0.01
	(57.82)	(66.84)	(71.26)	(78.06)	(79.93)
IV_t - RV_t	-0.07	-0.05	-0.03	-0.02	-0.01
	(24.32)	(19.73)	(12.76)	(10.54)	(9.69)
Adj. R^2 (%)	1.26	0.46	0.23	0.10	0.05
	Dependent Variable $R_{STRD,t}$				
Constant	-0.28	-0.29	-0.31	-0.33	-0.34
	(64.63)	(67.35)	(70.58)	(75.00)	(77.89)
IV_t - RV_t	-1.88	-1.65	-0.97	-0.54	-0.23
	(15.99)	(14.46)	(10.37)	(8.16)	(9.18)
Adj. R^2 (%)	0.31	0.25	0.11	0.06	0.00

Table C-4

Volatility risk premium regressions with a coefficient of relative risk aversion equal to 5.0

The table presents single-variable regressions of lagged the volatility risk premium on hold-to-maturity put option returns (R_{put}), put-delta-hedged portfolio returns (R_{DHput}) and straddle portfolio returns (R_{STRD}) over 1-, 3-, 6-, 9- and 12-month forecasting horizons. The variables ($IV_t - RV_t$), $R_{put,t}$, $R_{DHput,t}$, $R_{STRD,t}$ are defined in Table 1. The simulations are based on an economy with breaks and incomplete information under Bayesian learning. This table presents the results with a coefficient of relative risk aversion, α, at 5.0; however, additional regression analyses with different coefficients of risk aversion are reported in Appendix C. To adjust for heteroscedasticity and serial correlation, robust Newey-West (1987) t-statistics are used in the t-tests. The numbers in the table are the average estimates over 2,000 simulations; for each of these simulations, we generate 12 years (3,024 days) of daily dividends. The percentage of the simulations with significant statistics for the respective diagnostic tests is reported in parentheses at 5% significance on one-sided t-tests.

Monthly forecasting horizons	1	3	6	9	12
	Breaks - Inc. Inf. (Learning) and α = 5.0				
	Dependent Variable $R_{put,t}$				
Constant	-0.82	-0.88	-0.89	-0.90	-0.90
	(96.17)	(97.21)	(98.26)	(98.78)	(98.61)
$IV_t - RV_t$	-0.41	-0.26	-0.23	-0.20	-0.18
	(9.93)	(9.93)	(9.93)	(11.32)	(12.72)
Adj. R^2 (%)	0.96	0.46	0.33	0.26	0.12
	Dependent Variable $R_{DHput,t}$				
Constant	-0.02	-0.03	-0.05	-0.06	-0.07
	(45.99)	(83.80)	(93.73)	(95.30)	(95.64)
$IV_t - RV_t$	-0.18	-0.14	-0.09	-0.06	-0.03
	(100.00)	(95.82)	(68.29)	(35.37)	(17.60)
Adj. R^2 (%)	43.95	26.58	14.16	7.78	4.54
	Dependent Variable $R_{STRD,t}$				
Constant	-0.69	-0.72	-0.76	-0.79	-0.82
	(90.77)	(94.25)	(96.34)	(98.26)	(98.78)
$IV_t - RV_t$	-0.69	-0.59	-0.48	-0.38	-0.30
	(44.95)	(41.29)	(35.37)	(28.22)	(26.48)
Adj. R^2 (%)	3.82	2.89	2.00	1.38	0.95

Table C-5

Monthly return regression with a coefficient of relative risk aversion equal to 0.5

The table reports single-variable and multivariable regressions of one-month lagged predictor variables on hold-to-maturity put option contract returns R_{put} (Panel A), put-delta-hedged portfolio returns R_{DHput} (Panel B) and straddle portfolio returns R_{STRD} (Panel C). The simulations are based on an economy with breaks and incomplete information under Bayesian learning. This table presents the results with a coefficient of relative risk aversion, α, at 0.5; however, additional regression analyses with different coefficients of risk aversion are reported in Appendix C. The variables $(IV_t - RV_t)$, $R_{put,t}$, $R_{DHput,t}$, $R_{STRD,t}$, IV_t, RV_t, $Slope_{Mon,t}$ $Slope_{Mat,t}$ are defined in Table 1. The dividend yield, $DivYield_t$, is implicitly obtained from the call-put parity relationship of the European put and call option contracts (at-the-money one-month to maturity contracts). $R_{m,t}$ is the excess return on the stock (the stock price is calculated with Equation (8)). To adjust for heteroscedasticity and serial correlation, robust Newey-West (1987) t-statistics are used in the t-tests. The numbers in the table are the average estimates over 2,000 simulations; for each of these simulations, we generate 12 years (3,024 days) of daily dividends. The percentage of the simulations with significant statistics for the respective diagnostic tests is reported in parentheses at 5% significance on two-sided t-tests.

Panel A.	Monthly return regression Breaks - Inc. Inf. (Learning) and α = 0.5											
	Dependent Variable $R_{put,t}$											
Constant	-0.26	0.79	-0.36	-0.33	-0.65	-0.20	-0.39	0.05	1.18	-0.13	0.11	-0.21
	(22.62)	(6.46)	(10.88)	(61.39)	(32.31)	(6.80)	(94.90)	(8.33)	(7.48)	(7.82)	(8.33)	(8.84)
IV_t - RV_t	-3.93							-3.90				
	(7.82)							(7.65)				
IV_t		-12.79							-13.03			
		(11.56)							(11.90)			
RV_t			-0.50							-0.54		
			(6.12)							(5.78)		
$Slope_{Mon.t}$				-1.04							-1.11	
				(8.50)							(9.35)	
$Slope_{Mat.t}$					11.52							-2.23
					(12.76)							(6.63)
$DivYield_t$						-2.66		-4.39	-5.28	-3.30	-6.32	-4.37
						(5.78)		(7.48)	(7.65)	(7.65)	(7.65)	(8.16)
$R_{m.t}$							0.49	0.68	0.69	0.57	1.09	0.31
							(5.61)	(5.95)	(6.63)	(6.63)	(6.63)	(5.95)
Adj. R^2 (%)	0.38	0.95	0.01	0.27	0.52	0.01	-0.02	0.40	0.98	0.03	0.34	0.34

Panel B.	Monthly return regression Breaks - Inc. Inf. (Learning) and α = 0.5											
	Dependent Variable $R_{DHput.t}$											
Constant	-0.01	0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.01	0.01	-0.01	-0.01	-0.01
	(43.03)	(21.26)	(26.02)	(90.65)	(10.54)	(12.59)	(100.00)	(12.24)	(8.33)	(11.73)	(11.90)	(10.03)
IV_t - RV_t	-0.07							-0.07				
	(17.18)							(16.67)				
IV_t		-0.22							-0.22			
		(38.78)							(37.59)			
RV_t			-0.01							-0.01		
			(6.80)							(8.16)		
$Slope_{Mon.t}$				-0.02							-0.02	
				(17.86)							(17.01)	
$Slope_{Mat.t}$					-0.35							-0.37
					(36.39)							(63.78)
$DivYield_t$						0.08		0.08	0.05	0.09	0.06	0.12
						(9.86)		(10.54)	(9.86)	(10.88)	(9.86)	(10.20)
$R_{m.t}$							0.00	0.00	0.00	-0.01	0.00	-0.01
							(6.80)	(5.78)	(6.12)	(6.80)	(6.46)	(7.31)
Adj. R^2 (%)	1.26	2.57	0.09	1.20	2.42	0.06	0.08	1.31	2.58	0.23	1.26	5.50

Panel C.	Monthly return regression Breaks - Inc. Inf. (Learning) and α = 0.5											
	Dependent Variable $R_{STRD.t}$											
Constant	-0.28	0.18	-0.32	-0.32	-0.06	-0.58	-0.35	-0.52	-0.01	-0.59	-0.54	-0.42
	(53.91)	(8.84)	(27.38)	(91.33)	(13.10)	(12.76)	(100.00)	(10.37)	(7.65)	(11.73)	(11.56)	(12.24)
IV_t - RV_t	-1.88							-1.84				
	(10.03)							(9.69)				
IV_t		-5.77							-5.72			
		(17.52)							(16.50)			
RV_t			-0.49							-0.44		
			(6.63)							(6.12)		
$Slope_{Mon.t}$				-0.40							-0.38	
				(11.22)							(11.39)	
$Slope_{Mat.t}$					-12.94							-6.93
					(23.98)							(25.17)
$DivYield_t$						3.42		3.42	2.72	3.86	3.06	3.40
						(8.50)		(9.18)	(8.50)	(9.01)	(8.84)	(8.16)
$R_{m.t}$							-0.04	-0.12	-0.03	-0.17	-0.09	-0.14
							(6.97)	(7.14)	(6.80)	(7.48)	(7.31)	(7.14)
Adj. R^2 (%)	0.31	0.99	0.04	0.28	1.31	0.04	0.03	0.37	1.01	0.12	0.34	1.86

Table C-6

Monthly return regression with a coefficient of relative risk aversion equal to 5.0

The table reports single-variable and multivariable regressions of one-month lagged predictor variables on hold-to-maturity put option contract returns R_{put} (Panel A), put-delta-hedged portfolio returns R_{DHput} (Panel B) and straddle portfolio returns R_{STRD} (Panel C). The simulations are based on an economy with breaks and incomplete information under Bayesian learning. This table presents the results with a coefficient of relative risk aversion, α, at 0.5; however, additional regression analyses with different coefficients of risk aversion are reported in Appendix C. The variables $(IV_t - RV_t)$, $R_{put,t}$, $R_{DHput,t}$, $R_{STRD,t}$, IV_t, RV_t, $Slope_{Mon,t}$ $Slope_{Mat,t}$ are defined in Table 1. The dividend yield, $DivYield_t$, is implicitly obtained from the call-put parity relationship of the European put and call option contracts (at-the-money one-month to maturity contracts). $R_{m,t}$ is the excess return on the stock (the stock price is calculated with Equation (8)). To adjust for heteroscedasticity and serial correlation, robust Newey-West (1987) t-statistics are used in the t-tests. The numbers in the table are the average estimates over 2,000 simulations; for each of these simulations, we generate 12 years (3,024 days) of daily dividends. The percentage of the simulations with significant statistics for the respective diagnostic tests is reported in parentheses at 5% significance on two-sided t-tests.

Panel A.	Monthly return regression Breaks - Inc. Inf. (Learning) and α = 5.0											
	Dependent Variable $R_{put.t}$											
Constant	-0.82	-0.76	-0.96	-0.94	-0.76	-0.32	-0.96	-0.16	-0.11	-0.31	-0.24	-0.20
	(94.60)	(86.76)	(99.65)	(99.65)	(84.15)	(35.37)	(99.65)	(26.13)	(24.22)	(35.19)	(37.28)	(8.71)
IV_t - RV_t	-0.41							-0.60				
	(14.29)							(9.41)				
IV_t		-0.54							-0.73			
		(15.16)							(8.36)			
RV_t			0.01							-0.06		
			(6.27)							(5.75)		
$Slope_{Mon.t}$				0.26							-0.15	
				(32.23)							(23.87)	
$Slope_{Mat.t}$					-1.22							-2.19
					(13.59)							(6.79)
$DivYield_t$						-3.04		-2.77	-2.69	-3.11	-3.45	-4.55
						(17.94)		(8.54)	(8.54)	(13.94)	(3.83)	(8.19)
$R_{m.t}$							0.60	1.20	0.58	0.39	0.67	0.37
							(13.41)	(10.45)	(14.81)	(15.51)	(6.27)	(5.75)
Adj. R^2 (%)	0.96	1.22	-0.52	1.87	1.58	2.31	-0.16	3.12	3.27	1.75	2.70	0.35

Panel B.	Monthly return regression Breaks - Inc. Inf. (Learning) and α = 5.0											
	Dependent Variable $R_{DHput.t}$											
Constant	-0.02	0.00	-0.08	-0.09	0.01	-0.13	-0.08	0.00	0.03	-0.13	-0.03	-0.01
	(31.36)	(21.78)	(100.00)	(100.00)	(23.52)	(71.43)	(100.00)	(6.27)	(18.47)	(69.34)	(48.95)	(9.93)
IV_t - RV_t	-0.18							-0.19				
	(99.65)							(99.65)				
IV_t		-0.23							-0.23			
		(100.00)							(100.00)			
RV_t			-0.12							-0.12		
			(93.21)							(82.93)		
$Slope_{Mon.t}$				-0.10							-0.13	
				(82.06)							(89.72)	
$Slope_{Mat.t}$					-0.52							-0.37
					(99.30)							(63.24)
$DivYield_t$						0.24		-0.06	-0.09	0.24	-0.29	0.12
						(59.06)		(9.93)	(18.29)	(57.84)	(56.10)	(10.10)
$R_{m.t}$							-0.13	-0.05	0.03	-0.11	0.08	-0.01
							(36.06)	(31.71)	(15.16)	(23.00)	(39.72)	(7.32)
Adj. R^2 (%)	43.95	62.14	4.93	23.70	56.44	10.99	2.07	49.05	62.52	17.09	31.97	5.34

Panel C.	Monthly return regression Breaks - Inc. Inf. (Learning) and α = 5.0											
	Dependent Variable $R_{STRD.t}$											
Constant	-0.69	-0.63	-0.92	-0.95	-0.64	-1.31	-0.93	-0.77	-0.72	-1.31	-1.18	-0.42
	(86.41)	(73.17)	(100.00)	(100.00)	(76.66)	(75.96)	(100.00)	(49.30)	(44.60)	(73.69)	(58.89)	(12.20)
IV_t - RV_t	-0.69							-0.73				
	(23.34)							(19.34)				
IV_t		-0.80							-0.82			
		(18.64)							(17.94)			
RV_t			-0.08							0.00		
			(16.55)							(8.01)		
$Slope_{Mon.t}$				-0.27							-0.19	
				(8.36)							(3.31)	
$Slope_{Mat.t}$					-1.70							-6.89
					(19.16)							(24.56)
$DivYield_t$						1.73		0.45	0.45	1.73	1.07	3.34
						(8.01)		(1.39)	(1.92)	(6.27)	(6.10)	(8.19)
$R_{m.t}$							-0.48	-0.36	0.01	-0.61	-0.27	-0.14
							(8.01)	(4.53)	(4.01)	(4.01)	(5.40)	(7.14)
Adj. R^2 (%)	3.82	4.59	-0.60	1.07	3.64	1.37	-0.42	3.90	4.46	0.43	1.60	1.82

Table C-7

Three-month return regressions with a coefficient of relative risk aversion equal to 0.5

The table reports single-variable and multivariable regressions of three-month lagged predictor variables on hold-to-maturity put option contract returns R_{put} (Panel A), put-delta-hedged portfolio returns R_{DHput} (Panel B) and straddle portfolio returns R_{STRD} (Panel C). The simulations are based on an economy with breaks and incomplete information under Bayesian learning. This table presents the results with a coefficient of relative risk aversion, α, at 0.5; however, additional regression analyses with different coefficients of risk aversion are reported in Appendix C. The variables $(IV_t - RV_t)$, $R_{put,t}$, $R_{DHput,t}$, $R_{STRD,t}$, IV_t, RV_t, $Slope_{Mon,t}$ $Slope_{Mat,t}$ are defined in Table 1. The dividend yield, $DivYield_t$ and $R_{m,t}$ are defined in Table 4. To adjust for heteroscedasticity and serial correlation, robust Newey-West (1987) t-statistics are used in the t-tests. The numbers in the table are the average estimates over 2,000 simulations; for each of these simulations, we generate 12 years (3,024 days) of daily dividends. The percentage of the simulations with significant statistics for the respective diagnostic tests is reported in parentheses at 5% significance on two-sided t-tests.

Panel A.	Three-month return regression Breaks - Inc. Inf. (Learning) and α = 0.5											
	Dependent Variable $R_{put,t}$											
Constant	-0.36	-0.04	-0.34	-0.40	-0.38	-0.05	-0.39	0.05	0.42	0.05	0.00	-0.07
	(32.65)	(8.84)	(10.71)	(73.47)	(19.22)	(7.31)	(93.71)	(5.44)	(4.76)	(6.97)	(6.80)	(5.61)
$IV_t - RV_t$	-0.86							-0.89				
	(6.29)							(5.44)				
IV_t		-3.80							-4.04			
		(8.84)							(8.16)			
RV_t			-0.84							-0.83		
			(4.93)							(4.59)		
$Slope_{Mon.t}$				0.13							0.09	
				(7.65)							(6.97)	
$Slope_{Mat,t}$					-0.45							-3.30
					(8.84)							(8.16)
$DivYield_t$						-4.84		-5.93	-6.22	-5.64	-5.64	-5.98
						(5.61)		(5.27)	(5.27)	(5.61)	(5.78)	(5.61)
$R_{m,t}$							0.18	0.40	0.38	0.39	0.45	0.20
							(5.27)	(3.74)	(5.10)	(3.91)	(3.91)	(4.59)
Adj. R^2 (%)	-0.04	0.12	-0.08	-0.08	0.06	0.00	-0.02	-0.09	0.08	-0.12	-0.12	0.16

Panel B.	Three-month return regression Breaks - Inc. Inf. (Learning) and α = 0.5											
	Dependent Variable $R_{DHput,t}$											
Constant	-0.01 (54.93)	0.00 (14.29)	-0.01 (28.40)	-0.01 (94.90)	0.00 (20.75)	0.00 (9.18)	-0.01 (100.00)	0.00 (7.48)	0.01 (10.71)	0.00 (9.52)	0.00 (7.14)	0.00 (9.69)
IV_t - RV_t	-0.05 (13.27)							-0.05 (13.95)				
IV_t		-0.14 (26.36)							-0.14 (28.40)			
RV_t			0.00 (6.80)							0.00 (7.14)		
$Slope_{Mon,t}$				-0.01 (11.90)							-0.01 (11.90)	
$Slope_{Mat,t}$					-0.21 (24.32)							-0.25 (44.39)
$DivYield_t$						-0.08 (7.48)		-0.09 (9.01)	-0.11 (9.18)	-0.08 (8.67)	-0.10 (8.50)	-0.07 (9.35)
$R_{m,t}$							0.00 (6.29)	0.00 (5.61)	0.01 (5.95)	0.00 (5.44)	0.00 (5.27)	0.00 (5.61)
Adj. R^2 (%)	0.46	1.09	0.01	0.40	0.97	0.11	-0.05	0.50	1.14	0.06	0.44	2.92

Panel C.	Three-month return regression Breaks - Inc. Inf. (Learning) and α = 0.5											
	Dependent Variable $R_{STRD,t}$											
Constant	-0.29	0.03	-0.35	-0.33	-0.20	-0.24	-0.35	-0.16	0.22	-0.24	-0.18	-0.25
	(53.91)	(10.20)	(30.10)	(94.22)	(23.47)	(10.37)	(100.00)	(10.20)	(8.33)	(10.03)	(8.84)	(10.88)
IV_t - RV_t	-1.65							-1.64				
	(10.03)							(10.03)				
IV_t		-4.11							-4.13			
		(17.86)							(17.18)			
RV_t			-0.01							0.02		
			(7.31)							(6.63)		
$Slope_{Mon,t}$				-0.30							-0.32	
				(10.88)							(9.69)	
$Slope_{Mat,t}$					-6.82							-4.37
					(17.35)							(21.77)
$DivYield_t$						-1.59		-1.96	-2.65	-1.60	-2.16	-1.39
						(8.33)		(9.01)	(8.50)	(9.01)	(9.18)	(9.35)
$R_{m,t}$							-0.05	0.04	0.12	0.02	0.05	-0.02
							(5.27)	(5.78)	(5.27)	(5.95)	(4.76)	(5.27)
Adj. R^2 (%)	0.25	0.58	0.03	0.19	0.59	0.09	-0.07	0.26	0.60	0.04	0.18	1.06

Table C-8

Three-month return regressions with a coefficient of relative risk aversion equal to 5.0

The table reports single-variable and multivariable regressions of three-month lagged predictor variables on hold-to-maturity put option contract returns R_{put} (Panel A), put-delta-hedged portfolio returns R_{DHput} (Panel B) and straddle portfolio returns R_{STRD} (Panel C). The simulations are based on an economy with breaks and incomplete information under Bayesian learning. This table presents the results with a coefficient of relative risk aversion, α, at 5.0; however, additional regression analyses with different coefficients of risk aversion are reported in Appendix C. The variables $(IV_t - RV_t)$, $R_{put,t}$, $R_{DHput,t}$, $R_{STRD,t}$, IV_t, RV_t, $Slope_{Mon,t}$ $Slope_{Mat,t}$ are defined in Table 1. The dividend yield, $DivYield_t$ and $R_{m,t}$ are defined in Table 4. To adjust for heteroscedasticity and serial correlation, robust Newey-West (1987) t-statistics are used in the t-tests. The numbers in the table are the average estimates over 2,000 simulations; for each of these simulations, we generate 12 years (3,024 days) of daily dividends. The percentage of the simulations with significant statistics for the respective diagnostic tests is reported in parentheses at 5% significance on two-sided t-tests.

Panel A.	Three-month return regression Breaks - Inc. Inf. (Learning) and α = 5.0											
	Dependent Variable $R_{put,t}$											
Constant	-0.88	-0.84	-0.96	-0.94	-0.82	-0.41	-0.96	-0.27	-0.21	-0.41	-0.53	-0.07
	(96.17)	(89.55)	(99.65)	(99.65)	(89.20)	(43.38)	(99.65)	(32.58)	(30.66)	(43.03)	(43.73)	(5.75)
IV_t - RV_t	-0.26							-0.35				
	(8.54)							(6.27)				
IV_t		-0.34							-0.39			
		(11.67)							(7.49)			
RV_t			0.01							0.10		
			(5.57)							(3.83)		
$Slope_{Mon.t}$				0.40							0.17	
				(25.44)							(15.85)	
$Slope_{Mat,t}$					-0.89							-3.28
					(12.02)							(8.01)
$DivYield_t$						-2.63		-2.70	-2.85	-2.62	-2.02	-5.89
						(15.33)		(11.85)	(12.54)	(15.16)	(7.32)	(5.75)
$R_{m,t}$							0.76	1.12	0.88	1.18	0.60	0.17
							(9.41)	(8.89)	(9.76)	(11.32)	(7.32)	(4.53)
Adj. R^2 (%)	0.46	0.67	-0.42	1.13	0.98	1.76	-0.08	2.01	2.10	1.33	1.65	0.15

Panel B.	Three-month return regression Breaks - Inc. Inf. (Learning) and α = 5.0											
	Dependent Variable $R_{DHput,t}$											
Constant	-0.03	-0.02	-0.08	-0.09	-0.01	-0.12	-0.08	-0.02	0.00	-0.12	-0.04	0.00
	(70.56)	(42.86)	(100.00)	(100.00)	(29.44)	(67.60)	(100.00)	(10.80)	(10.63)	(67.77)	(41.29)	(9.58)
IV_t - RV_t	-0.14							-0.14				
	(89.90)							(91.99)				
IV_t		-0.17							-0.17			
		(99.13)							(98.61)			
RV_t			-0.10							-0.10		
			(89.55)							(79.97)		
$Slope_{Mon,t}$				-0.08							-0.10	
				(77.35)							(83.97)	
$Slope_{Mat,t}$					-0.41							-0.25
					(98.43)							(44.43)
$DivYield_t$						0.19		-0.05	-0.09	0.19	-0.24	-0.07
						(53.31)		(13.07)	(16.20)	(55.92)	(48.78)	(9.41)
$R_{m,t}$							-0.09	-0.02	0.04	-0.05	0.08	0.00
							(35.89)	(34.67)	(14.98)	(18.99)	(38.15)	(5.75)
Adj. R^2 (%)	26.58	37.39	3.01	17.08	35.01	8.97	1.34	30.14	37.59	12.59	22.52	2.87

Panel C.	Three-month return regression Breaks - Inc. Inf. (Learning) and α = 5.0											
	Dependent Variable $R_{STRD,t}$											
Constant	-0.72	-0.67	-0.92	-0.95	-0.66	-1.24	-0.92	-0.74	-0.68	-1.24	-1.10	-0.24
	(90.77)	(82.75)	(100.00)	(100.00)	(81.36)	(76.83)	(100.00)	(54.01)	(50.35)	(75.96)	(59.06)	(10.80)
IV_t - RV_t	-0.59							-0.61				
	(18.47)							(20.56)				
IV_t		-0.69							-0.69			
		(23.87)							(28.05)			
RV_t			-0.09							0.01		
			(17.07)							(7.32)		
$Slope_{Mon,t}$				-0.23							-0.17	
				(8.19)							(3.31)	
$Slope_{Mat,t}$					-1.54							-4.35
					(25.61)							(21.43)
$DivYield_t$						1.42		0.11	0.05	1.43	0.70	-1.46
						(10.28)		(2.79)	(4.18)	(8.54)	(6.79)	(9.41)
$R_{m,t}$							-0.39	-0.28	0.11	-0.41	-0.14	0.00
							(4.88)	(6.45)	(2.61)	(2.79)	(5.57)	(5.40)
Adj. R^2 (%)	2.89	3.52	-0.60	0.74	3.04	1.09	-0.41	2.80	3.21	0.18	0.97	1.05

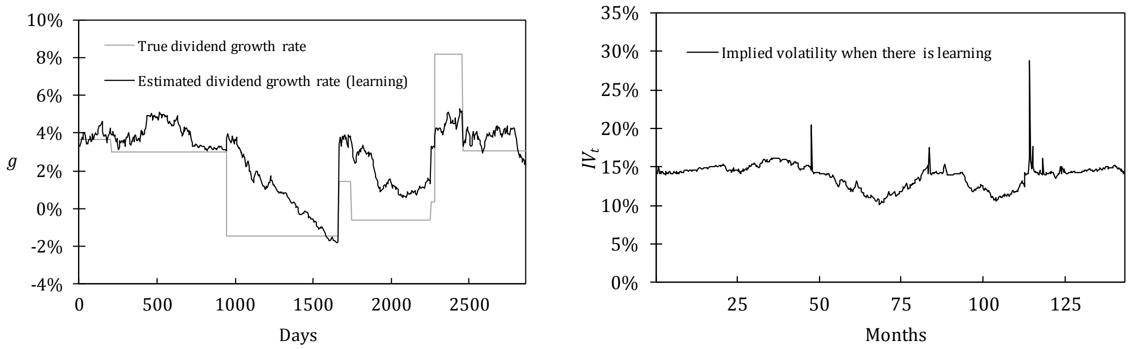

Figure 1. Evolutions of the mean dividend growth rate and the at-the-money short-term implied volatility when there is learning. The figure shows the outcome for one simulated path concerning the dynamics over a 12-year sample for the mean dividend growth rate (left hand side) and the at-the-money short-term implied volatility (right hand side) when there is incomplete information and rational learning. In this simulation, we set the coefficient of relative risk aversion at 0.2. In the case of learning the mean dividend growth rate is calculated using equation (6). IV_t is expressed on an annual basis and calculated as the average of the Black-Scholes' (1973) implied volatilities of a European put option contract and a European call option contract, where both contracts are at-the-money and with one-month to maturity.

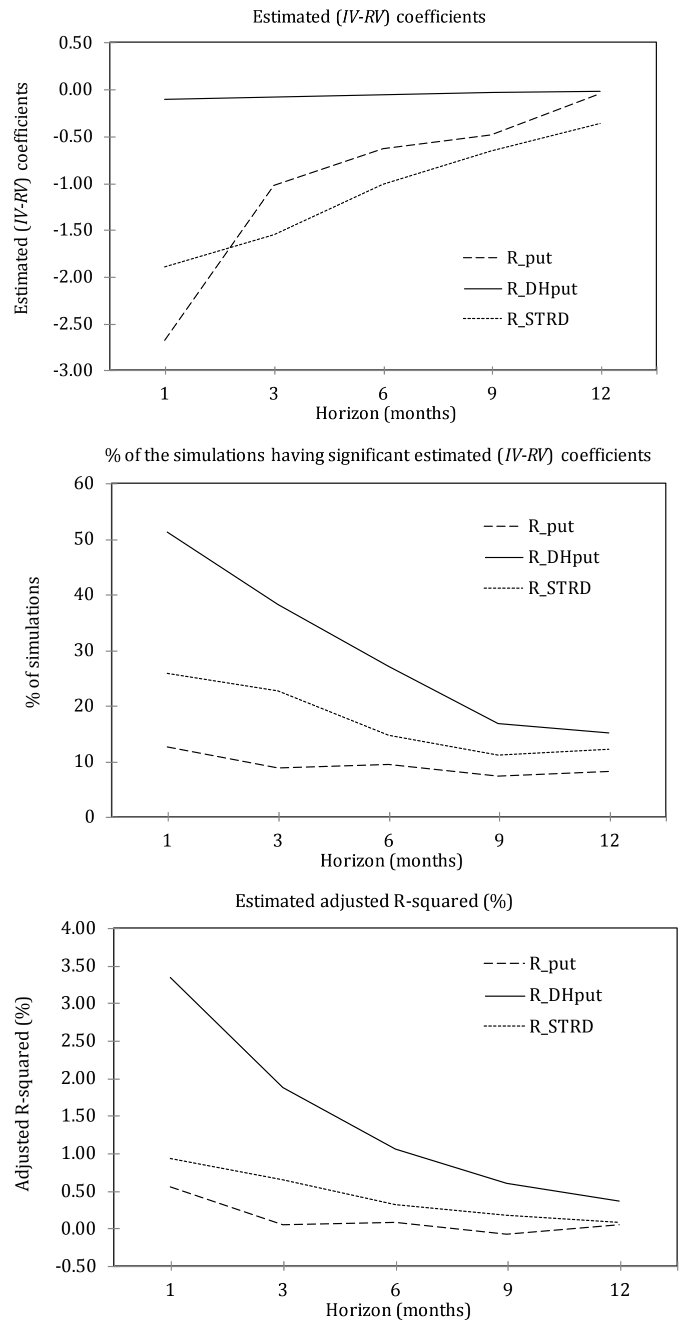

Figure 2. Estimated regression coefficients of the volatility risk premium, percentage of the simulations having significant statistics and adjusted R^2s. The figure shows the average estimated regression coefficients, the percentage of the simulations which have significant statistics for the slope coefficients and average adjusted R^2s. The figure presents single-variable regressions of the lagged $(IV - RV)$ on hold-to-maturity put option returns (R_{put}), put-delta-hedged portfolio returns (R_{DHput}) and straddle portfolio returns (R_{STRD}) over 1-, 3-, 6-, 9- and 12-month forecasting horizons. The variables $(IV - RV)$, R_{put}, R_{DHput} and R_{STRD} are defined in Table 1. The simulations are based on an economy with breaks and incomplete information under Bayesian learning. This figure presents the results with a coefficient of relative risk aversion, α, at 0.2. To adjust for heteroscedasticity and serial correlation, robust Newey-West (1987) t-statistics are used in the t-tests. The numbers in the figure are the average estimates over 2,000 simulations; for each of these simulations, we generate 12 years (3,024 days) of daily dividends. The percentage of the simulations with significant statistics for the respective diagnostic tests is reported in parentheses at 5% significance on one-sided t-tests.

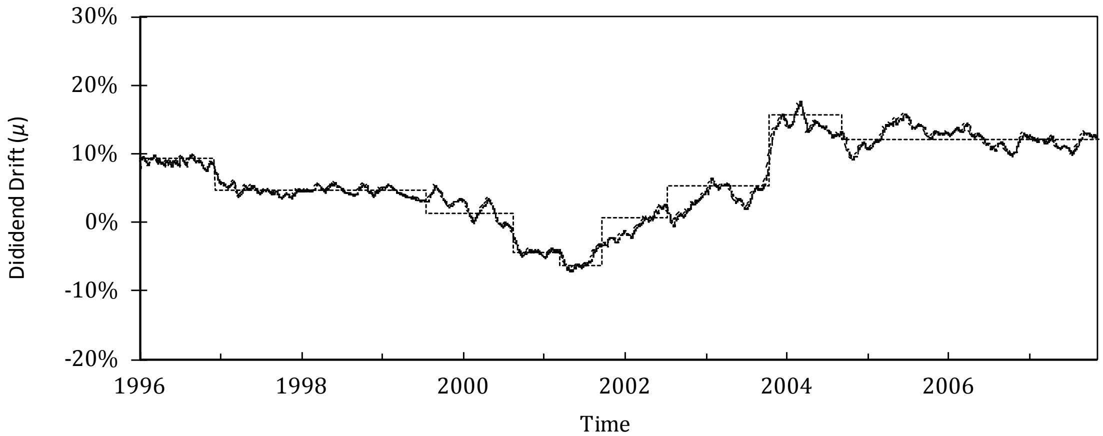

Figure B-1. Structural breaks in the drift of the random walk process. The solid line represents the dividend drift calculated with a rolling window of 125 trading days using log dividend-ratio from the S&P 500 index, which are deseasonalized and adjusted by the consumer price index to obtain real dividends between 1996 and 2007. The dotted line shows structural breaks in the dividend drift. Breaks are detected in December 1996, August 1999, September 2000, April 2001, October 2001, August 2002, November 2003, and October 2004.